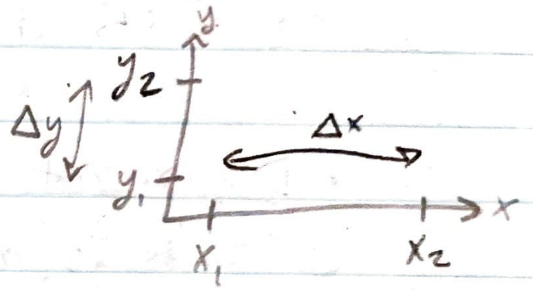
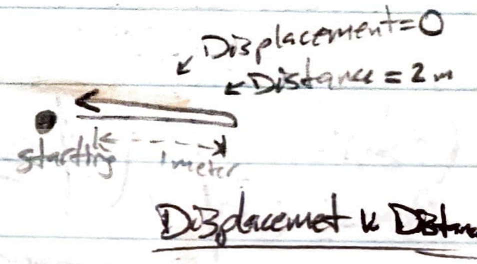
Kinematics in 1 Dimension

- Vectors / scalar Quantities
  - * Vector quantity → magnitude (size/units)
    - Magnitude & direction
  - * Scalar
    - Magnitude (Ex → Time)
- Displacement
  - change in position (Δx or Δy)

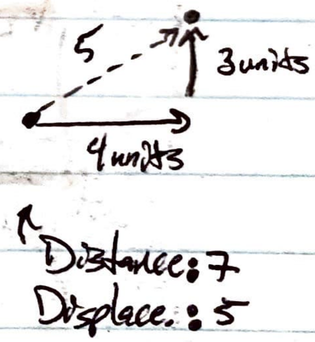


$$\boxed{\Delta x \equiv x_f - x_i}$$

- Frame of Reference
  - * starting pt of motion



- Distance
  - * How far object travelled
- Displacement takes Time
  - * Δx there is a Δt



- Velocity
  - * Speed & direction
  - * Meters per second
  - * Avg velocity → rate of displacement

$$\boxed{V_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_1}{t_f - t_1}}$$

→ $\left(V = \frac{\text{total distance}}{\text{total time}}\right)$

- Velocity
  - $V_{avg} = \frac{\Delta x}{t}$
  - V_{inst} = instantaneous $\vec{v}$
  - It's a vector

Vector
↓
magnitude & direction

- Displacement
  - Is a vector
  - Change in position

↑up = ⊕ ↓down = ⊖

$$\boxed{M = \frac{1}{2} a}$$

$$\boxed{\vec{a} = \partial M}$$

- Speed

$$\boxed{V_{(speed)} = \frac{d}{t}}$$

$\vec{V} \rightarrow$ vector

SI: m/s

- Acceleration (in 1D)
  - rate of change of the velocity

$$\boxed{\vec{a} = \frac{\Delta \vec{V}}{\Delta t}}$$

$\rightarrow$ slope of $\vec{V}$ v. T graph

*velocity is a vector (magnitude & direction)

*Change in direction is also acceleration

* Acceleration is a vector
  - direction matters
  - SI units: meters per second per second (m/s^2)
    - ↳ "velocity changes ((Blank) m/s) every second"

Motion w/ constant acceleration

d or $\Delta x \rightarrow$ displacement

$V_0 \rightarrow$ initial velocity (or V_i)

$V_f \rightarrow$ final velocity

$a \rightarrow$ acceleration

$\Delta t \rightarrow$ time interval

The Big 5 variables

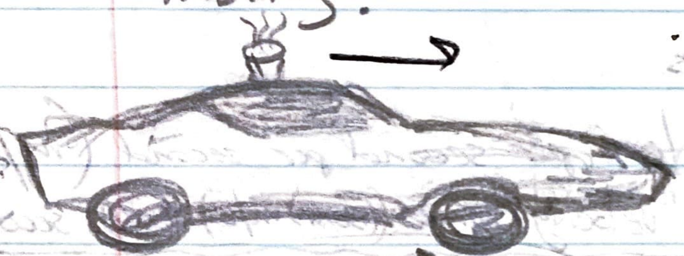
$$\boxed{V_f = V_0 + a\Delta t}$$

Ex.1 What is length of runway be for a plane to reach take off $\vec{V}$ of 75 m/s if $a = 2 m/s^2$?

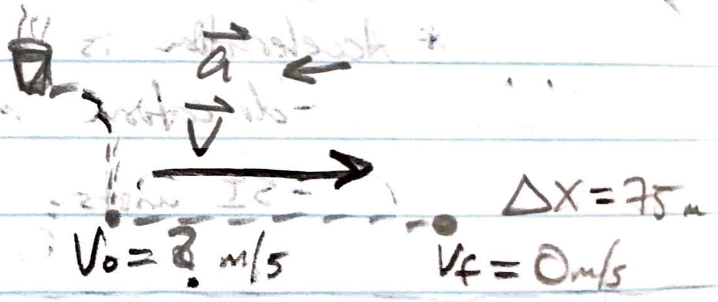
$a = 2 m/s^2$
$\Delta x = ?$
$V_0 = 0 m/s$
$V_f = 75 m/s$
$t = N/A$

$$(V_f)^2 = (V_0)^2 + 2a(\Delta x)$$
$$(75 m/s)^2 = 0^2 + 2(2)(\Delta x)$$
$$\boxed{\Delta X = 1406.25 \text{ meters}}$$

Ex.2 Cup of coffee falls off car & slides 75 m. Friction slows it down at $6 m/s^2$. How fast was car moving?



$\vec{V}_{car} = \vec{V}_{cup}$



$a = -6 m/s^2$

$$V_f^2 = V_0^2 + 2a\Delta x$$
$$0^2 = V_0^2 + 2(-6)(75)$$
$$\sqrt{900} = \sqrt{V_0^2}$$
$$\vec{V}_0 = 30 m/s$$

*To find t →

$$\Delta X = \frac{1}{2}(V_0 + V_f)t$$
$$75 = 0.5(30)t$$
$$t = 5 \text{ seconds}$$

$$\boxed{\Delta x = v_0 \Delta t + \frac{1}{2} a (\Delta t)^2}$$

$$\boxed{(v_f)^2 = (v_0)^2 + 2 a \Delta x}$$ ← (timeless equation)

1. constant acceleration
2. Unknown is one of the Big 5
3. 3 of the Big 5 are given

$$\boxed{\Delta x = \frac{1}{2} (v_0 + v_f) t}$$

$\boxed{\text{Ex:}}$ Person starts @ rest uniformly accelerates at a rate of 0.5 m/s^2. What is velocity after it traveled 4.75 m.

$a = .5\ m/s^2$

$\Delta x = 4.75\ m$

$v_0 = 0.0\ m/s$ ← "from rest"

$v_f = ?$

$$(v_f)^2 = (v_0)^2 + 2 a \Delta x$$

$$v_f = \pm \sqrt{2 a \Delta x}$$

$$v_f = \pm \sqrt{2(.5)4.75}$$

$$v_f = \pm 2.2\ m/s$$

$$v_f = 2.2\ m/s \text{ East}$$

Ex: Race car starts from rest, accelerates @ a rate of $4.9\ m/s^2$
What is speed after it travelled 200 meters?

$V_0 \rightarrow 0$
$V_f \rightarrow$ Find
$\Delta x \rightarrow 200m$
$a \rightarrow 4.9\ m/s^2$
$t \rightarrow ?$

$$V_f^2 = (V_0)^2 + 2a\Delta x$$
$$V_f^2 = 0^2 + 2(4.9\ m/s^2)(200\ m)$$
$$\sqrt{V_f^2} = \pm\sqrt{1960\ m/s^2}$$
$$V_f = +\ 44.3\ m/s$$

Ex: Hammer drops 6 meters vertically in 2.7 seconds what is a?

$V_f =$ Do not need to find
$V_0 = 0$
$\Delta y = 6\ m$
$a =$ Find
$t = 2.7s$

Horizon. → Δx
Vertical → Δy
y ↑ → x

$$\Delta y = V_0(\Delta t) + \frac{1}{2}a(\Delta t)^2$$
$$\Delta y = \cancel{0(\Delta t)} + \frac{1}{2}a(2.7s)^2$$
$$\Delta y = \frac{1}{2}a(2.7s)^2$$
$$a = \frac{2\Delta y}{t^2} \rightarrow a = \frac{2(6m)}{(2.7s)^2} \rightarrow 1.6\ m/s^2$$

Ex: Car traveling @ 15 m/s accelerates to 21 m/s in 12 seconds
Find total distance traveled by car in this 12 sec time interval.

$V_0 \rightarrow 15\ m/s$
$V_F \rightarrow 21\ m/s$
$a \rightarrow ?$ (Do not need)
$\Delta t \rightarrow 12$ seconds
$\Delta X \rightarrow$ Find

$$\Delta X = \frac{1}{2}(V_0 + V_f)t$$
$$\Delta X = \frac{1}{2}(15 + 21)12$$
$$\Delta X = 216 \text{ meters}$$

1 foot = 0.3048 m

To find meters:

(ft)(0.3048) = meters

Misc.

- Speed conversions

$\boxed{\text{Ex:}}$ 25 m/s → mph

$$25\frac{\cancel{m}}{s}\left(\frac{1\,\cancel{km}}{1000\,\cancel{m}}\right)\left(\frac{1\,mi}{1.61\,\cancel{km}}\right) = 0.0155\,\frac{mi}{s}$$

$$0.0155\frac{mi}{\cancel{s}}\left(\frac{60\,\cancel{s}}{1\,\cancel{min}}\right)\left(\frac{60\,\cancel{min}}{1\,hr}\right) = \boxed{55.9 \text{ miles per hour}}$$

$\boxed{\begin{array}{c} 1\text{ mile} = 1.61\text{ km} \\ 1\text{ km} = 1000\text{ m} \end{array}}$

$\boxed{\text{Ex:}}$ mph → mps

65 mph → mps

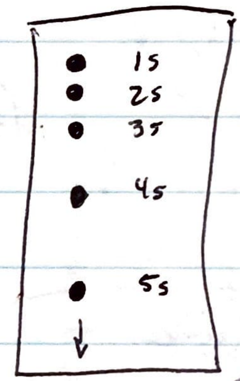
$$65\frac{\cancel{mi}}{hr}\left(\frac{1.61\,\cancel{km}}{1\,\cancel{mi}}\right)\left(\frac{1000\,m}{1\,\cancel{km}}\right) = 104{,}650\ \frac{m}{hr}$$

$$104{,}650\frac{m}{\cancel{hr}}\left(\frac{1\,\cancel{hr}}{60\,\cancel{min}}\right)\left(\frac{1\,\cancel{min}}{60\,s}\right) = \boxed{29.1\ m/s}$$

- Acceleration due to Gravity

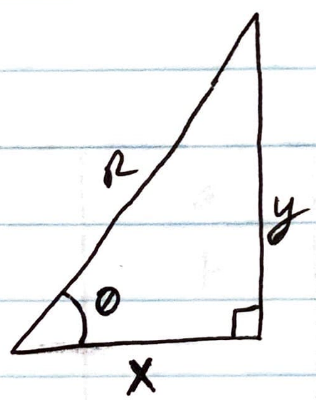
$g = -9.80\ m/s^2$

(symbol is g)



Near Earth's Surface

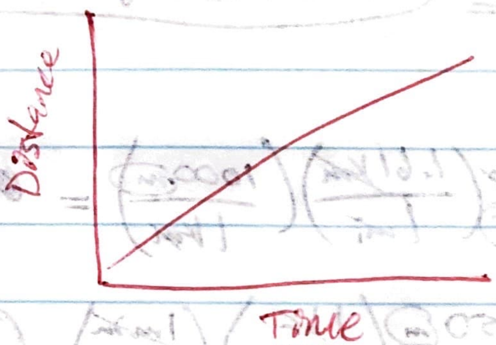
- Trig



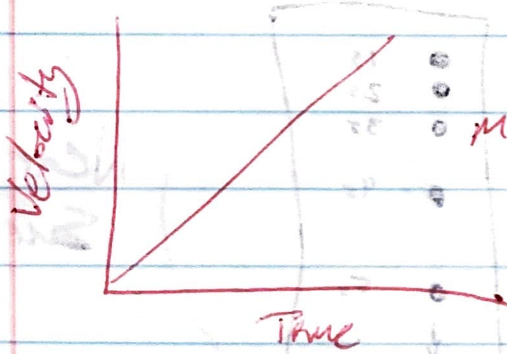
$$\boxed{\begin{array}{l} \sin\theta = \dfrac{y}{R} \\ \cos\theta = \dfrac{X}{R} \\ \tan\theta = \dfrac{y}{X} \end{array}}$$

• Constant acceleration ($\vec{a}$) means that the $\vec{v}$ v. t graph ~~is~~ linear

• To find instantaneous $\vec{v}$ → take tangent of point of curve



$m =$ velocity

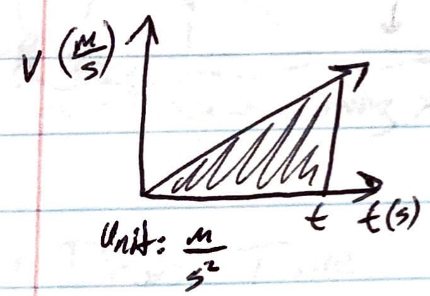


$m =$ acceleration

- Velocity from Graphs
  - Position vs Time graphs
  - $\vec{v}$ average = slope of line joining x_i & x_f
  - straight line (constant velocity)

- Instantaneous Velocity → indicates what is happening at any one instant

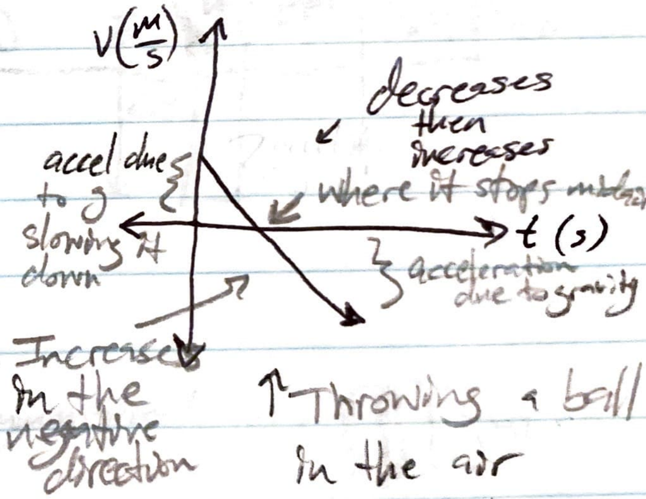
- Instantaneous Acceleration → (slope of tangent to the curve)



$A_\blacktriangle = \frac{1}{2} bh$

$= \frac{1}{2}\left(\frac{s}{1}\right)\left(\frac{m}{s}\right)$

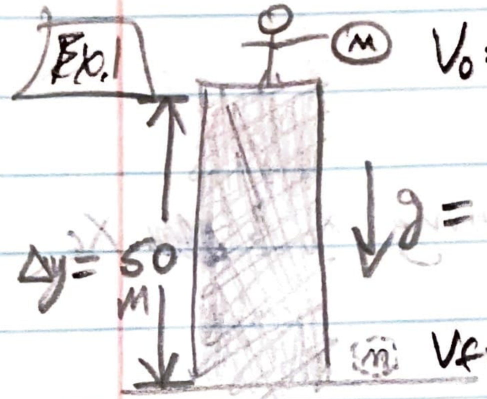
Displacement $= \frac{1}{2} m$



V_i	a	Motion
+	+	speeding up
−	−	speeding up
+	−	slowing down
−	+	slowing down
+ or −	0	constant vel.
0	+ or −	speeding up from rest
0	0	@ rest

Velocity / Acceleration relationship

Ex.1



$V_0 = 0$ m/s
$g = 9.8$ m/s
$\Delta y = 50$ m
$V_f = ?$
$t = N/A$

$V_f^2 = V_0^2 + 2a\Delta y$

$V_f^2 = 0^2 + 2(-9.8)(-50)$

$\sqrt{V_f} = \sqrt{980}$

$\boxed{V_f = 31.3 \text{ m/s}}$

NOTE: Acceleration Due to Gravity is always NEGATIVE

-9.81 m/s² ↓ Downwards towards center of Earth

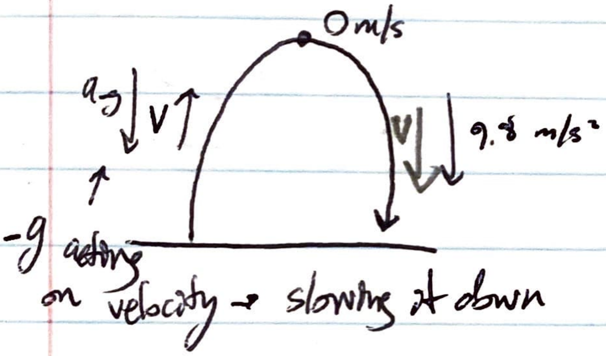
Free Fall

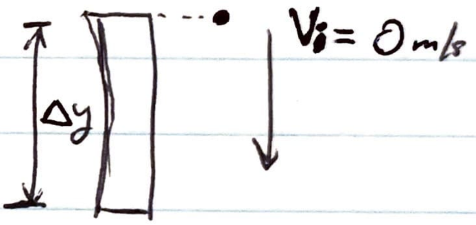
- <u>Definition</u> → moving freely w/ gravity the only thing influencing object
  $-g =$ is constant
- Objects will accelerate downwards
  - "acceleration due to gravity" (or "Free fall acceleration")
  - symbol: $\boxed{a_g}$ or $\boxed{g \text{ or } -g}$ Ex: g m/s^2

gravity is only influence

$a_g = -9.80\ m/s^2$ downwards

- Ignore air resistence for quantitative analysis





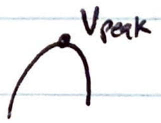
- Drop Time
  - time it takes to fall
  - If $\Delta y = 0$, the trip is symmetrical →

$$\boxed{t_{up} = t_{down}}$$

$$\boxed{V_f = -V_i}$$

$$\boxed{V_i = 0}$$

$$\boxed{V_{peak} = 0\ m/s}$$



$$\boxed{\Delta x = \frac{1}{2} g t^2}$$

g = acceleration due to gravity
Δx = distance

Solving Star-Problems

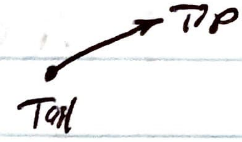
- On a seperate sheet of paper

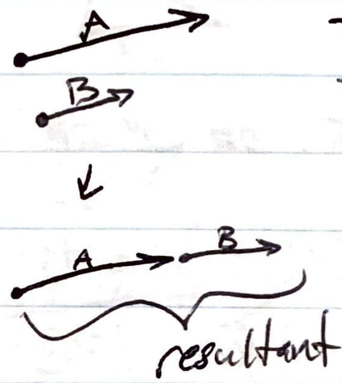
1. Identify (key concept of question)
2. Diagram
3. Givens/Unknowns
4. Equation/solve
5. Answer (must be boxed)
6. Check

Ch. 3
Vectors

- Adding vectors → resultant (is answer)
- Resolving a vector → breaking it up

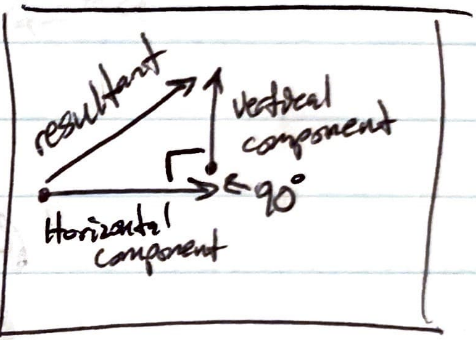
* Vector → magnitude/direction

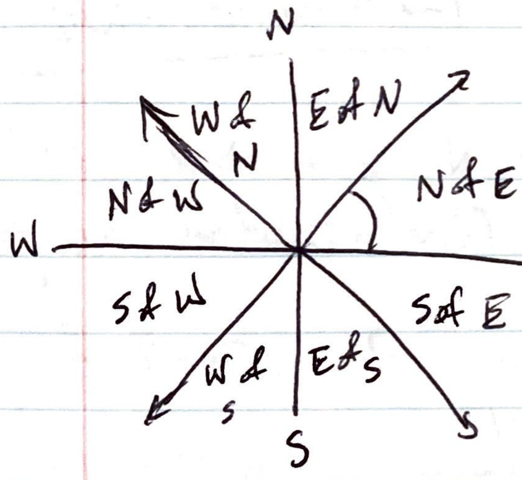




The lengths are magnitudes
The direction in which it is pointed

← Tip to tip

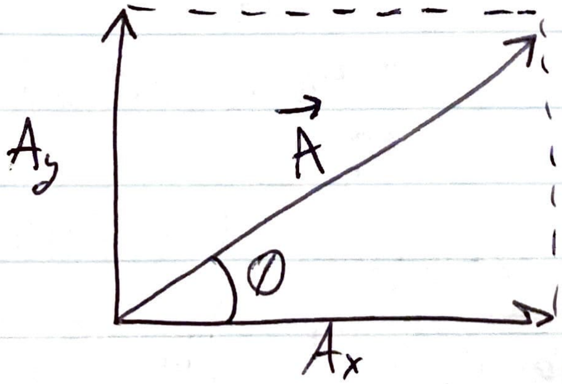




$\sin \theta = \frac{opp}{hyp}$

$\cos \theta = \frac{adj}{hyp}$

$\tan \theta = \frac{opp}{adj}$



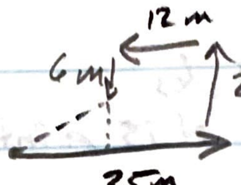
$$\boxed{A = \sqrt{a_x^2 + a_y^2}}$$

$$\boxed{\tan \theta = \frac{a_y}{a_x}}$$

$$\boxed{A_x = A \cos \theta}$$

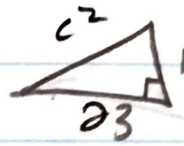
$$\boxed{A_y = A \sin \theta}$$

Ex: A bear travels 35 m (E) then 20 m (N), 12 m (W) then 6 m (S). Find displacement.



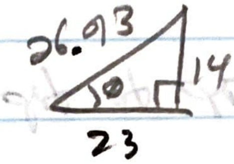
$35 - 12 = 23$ m

$20 - 6 = 14$ m



$14^2 + 23^2 = c^2$

$c = 26.93$ m



$\tan Ø = \frac{14}{23}$

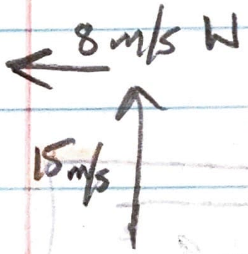
$\tan^{-1}(\tan)Ø = \tan^{-1}\left(\frac{14}{23}\right)$

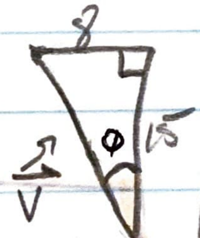
$\boxed{Ø = 31.33°}$

"Resultant displacement" →

$\boxed{\Delta x = 26.93 \text{ m N/E}}$

Ex: Boat moves w/ $\vec{V}$ of 15 m/s N in a river which flows w/ $\vec{V} = 8$ m/s W. Find resultant velocity.





$8^2 + 15^2 = c^2$

$c = 17$

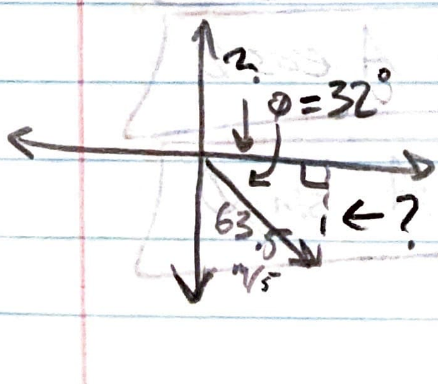
$\boxed{17 \text{ m/s @ } 28.1°}$

$\tan Ø = \frac{8}{15}$

$Ø = \tan^{-1}\left(\frac{8}{15}\right)$

$Ø = 28.1°$

Ex. Plane moves w/ $\vec{V} = 63.5$ m/s @ 32° S of East. Find Horizontal / Vertical $\vec{V}$.



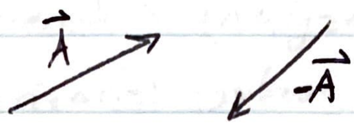
$\sin 32° = \frac{\text{opp}}{63.5}$

$\boxed{\text{Vertical } \vec{V} = 33.64 \text{ m/s}}$

$\cos 32° = \frac{\text{adj}}{63.5}$

$\boxed{\text{Horizontal } \vec{V} = 53.85 \text{ m/s}}$

- Negative Vectors
    - Same magnitude in opposite directions (180°)



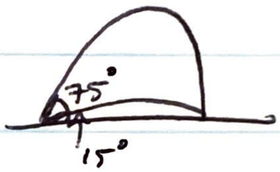
← Have to keep same magnitude

- Multiplying or Dividing by a Scalar
    - multiply magnitude
    - θ stays the same

Projectile Motion

- Definition: launched, freefall, path → trajectory
  - parabola (no air resistance)
- 2 Dimensions (analyze separately)
  - horizontal (sideways)
  - vertical (freefall / up & down)

- Range → horizontal displacement (Δx)
- Complementary values →



complimentary ∡s will yeild same Δx

* Vertical / horizontal components are independent except for TIME

- Givens for every projectile problem

$a_x = 0$ (No horizontal acceleration)
$a_y = -9.8\ m/s^2$
$V_{oy} = 0\ m/s$

For all Q's

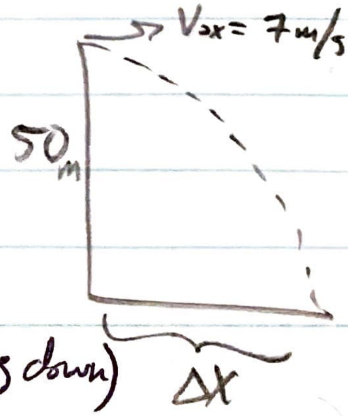
$$\boxed{\Delta x = V_{ox}\, t}$$

plus into that

Ex. 1 A ball is thrown horizon. w/ $V_o = 7\ m/s$ from a 50m building what is range?

H	V
$a_x = 0\ m/s^2$	$a_y = 9.8\ m/s^2$
$V_{ox} = 7\ m/s$	$V_{oy} = 0\ m/s$
$\Delta x = ?$	$\Delta y = -50\ m$
$t = 3.19$ sec	$t = ?$

negative (going down)



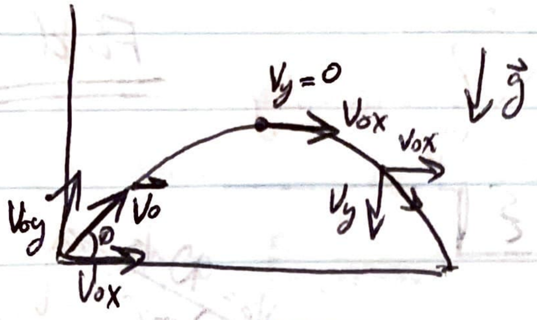
$\Delta y = \cancel{V_0 t} + \frac{1}{2} a_y t^2$

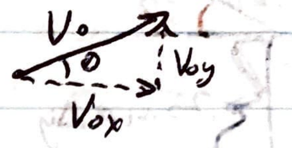
$t = \sqrt{\frac{2\Delta y}{a_y}}$

$t = 3.19$ sec

$\boxed{\Delta x = 22 m \text{ forward}}$

Angled Launches





$V_{ox} = V_o \cos\theta$

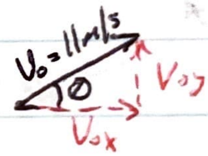
$V_{oy} = V_o \sin\theta$

If $\Delta y = 0$, $\boxed{t_{peak} = \frac{1}{2}(\text{roundtrip } t)}$ ← Because Path is Symmetrical

$V_y = 0$ @ peak

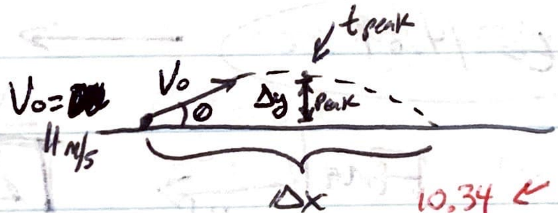
- There isn't horizontal acceleration so use $\boxed{\Delta x = V_{ox} t}$

Ex: Long jumper leaves ground at 11.0 m/s at an angle of 20° above horizontal.



Need to find:

- $t_{peak} = ?$
- $\Delta y_{peak} =$
- $\Delta x =$



$V_{ox} = V_o \cos\theta$

10.34 m/s ← $V_{ox} = 11(\cos(20°))$

$V_{oy} = V_o(\sin\theta)$

3.76 m/s ← $V_{oy} = 11(\sin(20))$

H	V
$V_{ox} = 10.34$ m/s	$V_{oy} = 3.76$ m/s
$a_x = 0$ m/s²	$a_y = -9.8$ m/s²
$t_{total} = ?$	$V_{fy \text{ or peak}} = 0$ m/s
	$t_{peak} = 0.38$ s

$V_f = V_o + at$

$t = \frac{V_f - V_o}{a}$

$\boxed{t_{peak} = 0.38 \text{ seconds}}$

$\Delta x = V_{ox} t + \frac{1}{2} a t^2$

$t_{total} = 2(t_{peak})$

$t = 0.76$ s

$\Delta x = 10.34(0.76) + \frac{1}{2}(0)(.76^2)$

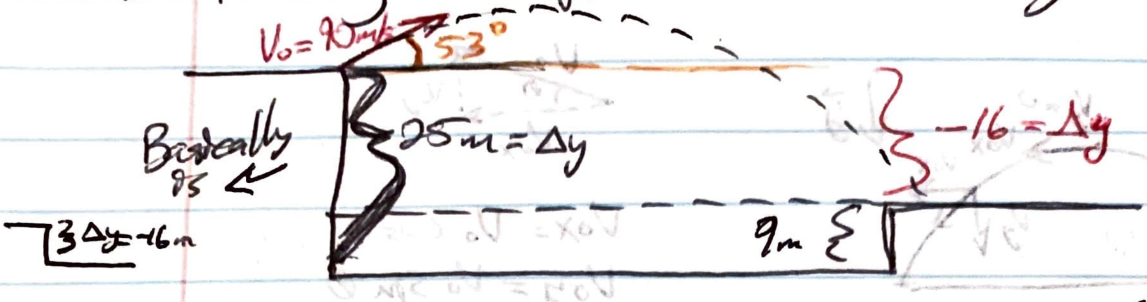
$\boxed{\Delta x = 7.86 \text{ meters}}$

$\Delta y_{peak} = V_{oy} t_{peak} + \frac{1}{2} a (t_{peak})^2$

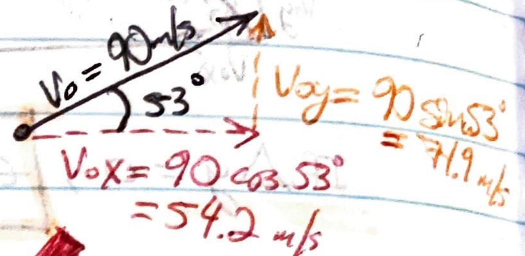
$\Delta y_{peak} = 3.76(0.38) + \frac{1}{2}(9.8)(0.38)^2$

$\boxed{\Delta y_{peak} = 0.71 \text{ m}}$

Finding Displacement of an Angled Launch Problem



Find Δx



NOTE: For launched problems, the V_{ox} or V_{oy} must be included in the equation (Not just V_0)

$$\Delta x = V_{ox}t + \tfrac{1}{2}at^2$$

$$\Delta y = V_{oy}t + \tfrac{1}{2}at^2$$

Values ~~[illegible]~~ subed in for V_{oy} & in the $\Delta y =$ ... eqn because of the -9.8 (vertical acceleration)

$$-16 = 71.9(t) + \tfrac{1}{2}(-9.8)(t^2)$$

$$-4.9t^2 + 71.9t + 16 = 0$$

$$t = \frac{-71.9 \pm \sqrt{71.9^2 - 4(\frac{1}{2}(-9.8))(16)}}{2(\frac{1}{2} \cdot -9.8)}$$

$$t = 14.89 \text{ sec}$$

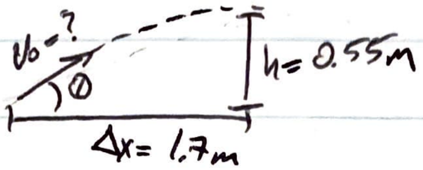
$$\Delta x = V_{ox}t + \tfrac{1}{2}at^2 \quad a_x = 0 \text{ m/s}^2$$

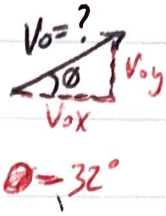
$$\Delta x = (90\cos 53)(14.89) + \cancel{\tfrac{1}{2}(0)(14.89)^2}$$

Final answer!!!

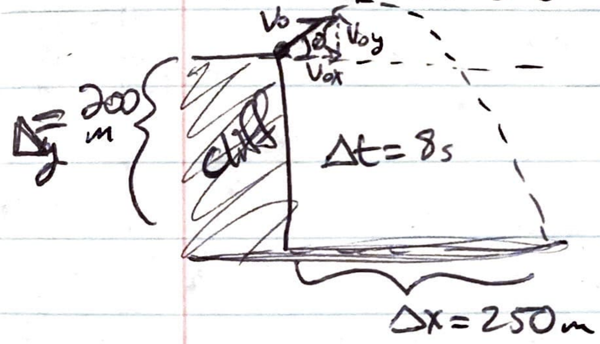
$$\boxed{\Delta x = 806.64}$$

$\boxed{\text{Ex.2}}$ Salmon starts 1.7 m from waterfall that is 0.55 m tall & jumps at ∡ of 32° Find minimum speed to reach waterfall?





$\boxed{\text{Ex.3}}$ Ball launched from 200 m cliff & lands 250 m away from cliff's base in 8 seconds. Find V_0 / θ



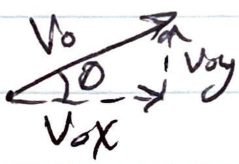
$$\Delta X = V_{ox}t + \frac{1}{2}at^2$$

$$\Delta X = V_{ox}t + \cancel{\frac{1}{2}(a_x = 0\,m/s^2)t^2}$$

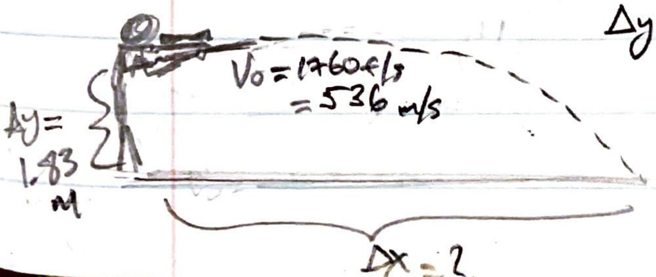
$$\Delta X = V_{ox}t$$

$$\frac{250}{8} = V_{ox} \qquad V_{ox} = 31.25\,m/s$$

$ft(0.3048) = m$



$\boxed{\text{Ex.4}}$ Rifle fired @ 1760 fps at shoulder height straight @ target. Find Δx



$$\Delta y = \cancel{V_{oy}t} + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{-1.83}{0.5(-9.8)}}$$

$$t = 0.61\,sec$$

$$\Delta x = V_{ox}t$$

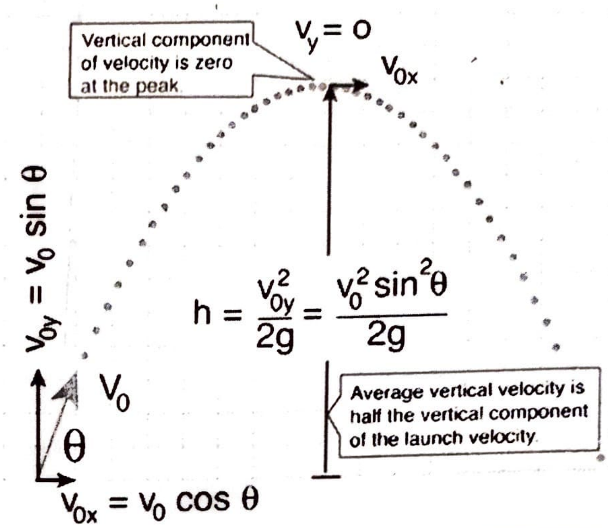
$$\Delta X = 536(0.61)$$

$$\Delta X = 327.8\text{ meters}$$

or $\boxed{1075.6\text{ ft}}$

$$\boxed{y_{max} = \frac{-V_{oy}^2}{2a}}$$

Height of Trajectory



The basic motion equation

$$y = \bar{v}_y t$$

can be used to find the height. The average vertical speed is:

$$\bar{v}_y = \frac{v_{0y} + 0}{2} = \frac{v_{0y}}{2}$$

The time at the peak is obtained by solving for the time at zero vertical speed:

$$0 = v_{0y} - g t_{peak}$$

This gives:

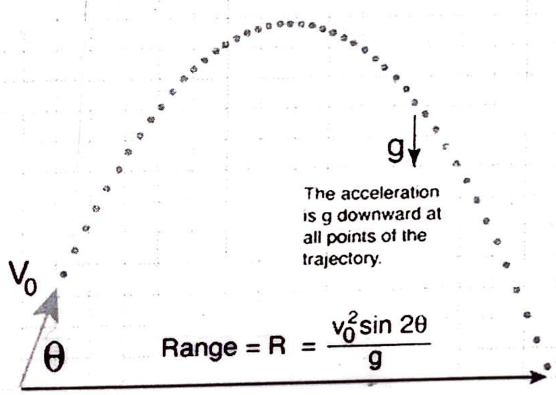
$$t_{peak} = \frac{v_{0y}}{g}$$

and substituting:

$$h = y_{peak} = \frac{v_{0y}^2}{2g}$$

Calculation

Range of Trajectory



The basic motion equation

$$x = v_{0x} t$$

can be used to find the range. By symmetry, the total time of flight is equal to twice the time at the peak:

$$t_{range} = 2t_{peak} = \frac{2v_{0y}}{g}$$

This gives:

$$R = \frac{2v_{0x} v_{0y}}{g}$$

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

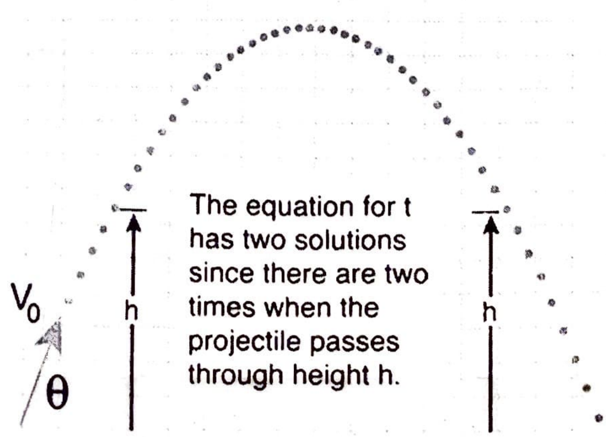
$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Calculation

using the trig identity:

$$\sin 2\theta = 2\sin\theta \cos\theta.$$

Time of Flight



h is also Δx

The basic motion equation

$$h = v_{0y} t - \frac{1}{2} g t^2$$

can be used to find the time of flight at height h, giving:*

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2h}{g}}$$

Note that there is no real solution if

$$\frac{2h}{g} > \frac{v_{0y}^2}{g^2} \quad \text{or } h > \frac{v_{0y}^2}{2g}$$

since such values of h are above the peak of the trajectory. For the value h=0:

$$t = 0 \text{ and } t = \frac{2v_{0y}}{g}$$

Calculation

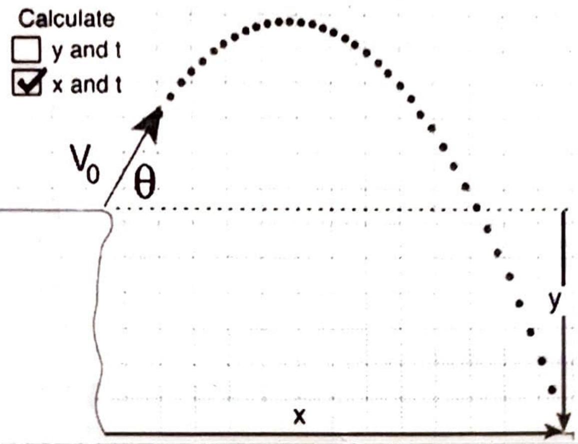
*quadratic formula

Where will it land?

The basic motion equations give the position components x and y in terms of the time. Solving for the horizontal distance in terms of the height y is useful for calculating ranges in situations where the launch point is not at the same level as the landing point.

$$x = v_{0x} t$$

$$y = v_{0y} t - \tfrac{1}{2} g t^2$$



Using the **quadratic formula** to solve for t gives two values of time for a given value of y:

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2y}{g}}$$

Substitution of the two time values gives the two values of x corresponding to a given height y.

Calculation

General Ballistic Trajectory

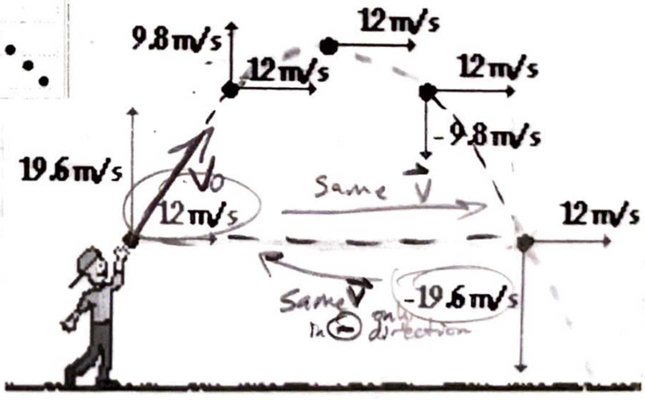
The motion of an object under the influence of gravity is determined completely by the acceleration of gravity, its launch speed, and launch angle provided air friction is negligible. The horizontal and vertical motions may be separated and described by the general motion equations for constant acceleration. The initial vector components of the velocity are used in the equations. The diagram shows trajectories with the same launch speed but different launch angles. Note that the 60 and 30 degree trajectories have the same range, as do any pair of launches at complementary angles. The launch at 45 degrees gives the maximum range.

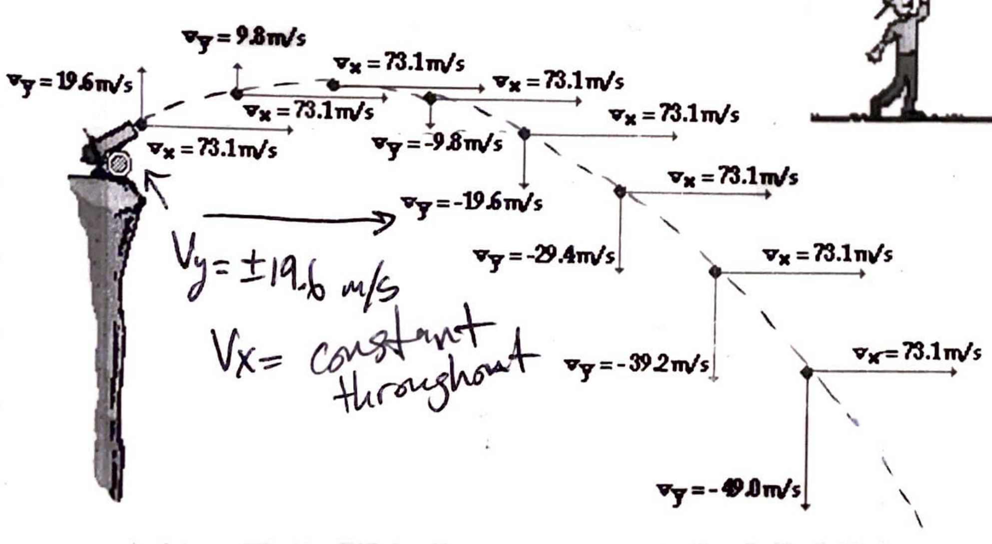
Horizontal Motion →

$$a_x = 0$$
$$V_x = V_{0x}$$
$$x = V_{0x} t$$

Vertical Motion ↑

$$a_y = -g$$
$$V_y = V_{0y} - gt$$
$$y = V_{0y} t - \tfrac{1}{2} g t^2$$





$V_y = \pm 19.6$ m/s

V_x = constant throughout

(V_{0x} is constant throughout)

Chapter 4 The Laws & Motions

- **Force** → a push or pull on an object
  - is a vector quantity SI: $\vec{F}$

- **Contact Force** → result from a physical contact between 2 objects EX: pulling a wagon, kicking a ball

- **Field Force** → describes a non-contact force acting on a particle at various positions in space EX: atoms/magnets

- **Normal Force** → (F_{norm}) is the support force exerted upon an object that is in contact with another stable object

  EX: Book on top of table; table is exerting upward force on the book in order to support weight of book

  ↳ Perpendicular to surface

- **Mass** → measure of an objects resistence to changes in the motion due to a force

- **Weight** → The force of gravity acting on an object

* **Mass** is related to how much stuff there is while **weight** is related to the pull of Earth's gravity upon that object (or its mass). The mass of an object (In Kg) will be the same no matter the location of the object in the universe. Mass is never altered by location, gravity, speed, or even other forces.

- Force is measured in Newtons
  $\vec{F}$ $\boxed{\vec{F} = ma}$

- Newton's 3 Laws of Motion

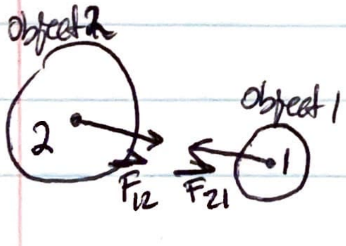
  ① An object moves w/ a velocity that is constant in magnitude & direction unless acted on by a nonzero net force.

  * Inertia ⇒ objects tendency to continue original state of motion

  ② The acceleration $\vec{a}$ of an object is directly proportional to the net force acting on it inversely proportional to its mass.

  $$\boxed{\sum \vec{F} = m\vec{a}} \qquad \vec{a} = \frac{F}{m}$$

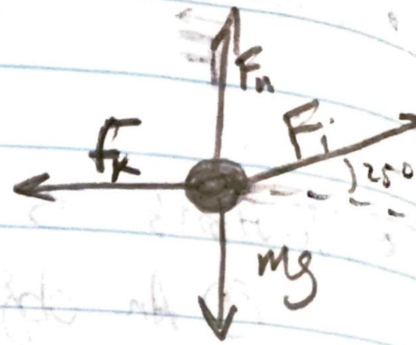
  ③ If object 1 & object 2 interact, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1.



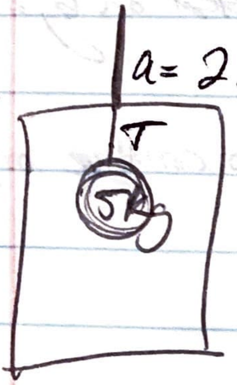
$$\vec{F}_{12} = -\vec{F}_{21}$$

Ex:

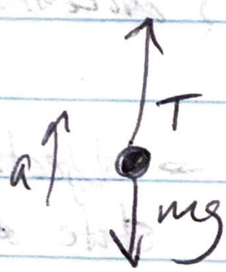
Mass = 35 Kg
Pull 185 N @ 25° above H
Friction = 119 N
$a = ?$



Ex:

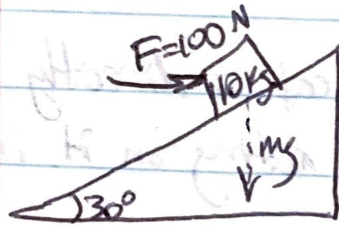


Find T

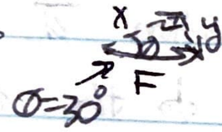


$F_{net} = ma$
$T - Mg = Ma$
$T - (5)(9.8) = (5)(2)$
$\boxed{T = 59\,N}$

Ex:



Find $\vec{a}$



$x = \vec{F}\cos\theta$
$x = 100\cos 30$
$x = 86.61\,N$

X component of mg is equal to $mg\sin\theta$

$X = 10(9.8)\sin 30$
$= 49$

$86.61 - 49 = F_{net}$
$= 37.6\,N$

Plug into $\Sigma F = m\vec{a}$

$\Sigma F = m\vec{a}$
$37.6 = 10(a)$
$\boxed{a = 3.76\,m/s^2}$

$F_g \equiv \vec{W} \equiv mg$

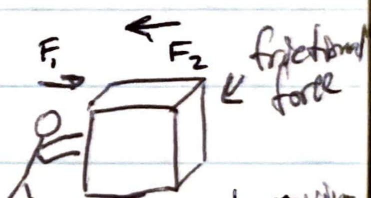
* object is in equilibrium even when object moves as long $\vec{a} = 0$ (must be in a straight line)

<u>Equilibrium</u> → Net Force $(\Sigma \vec{F}) = 0$
$\vec{a} = 0$

$\vec{F}_N$ = normal force $(\vec{n})$

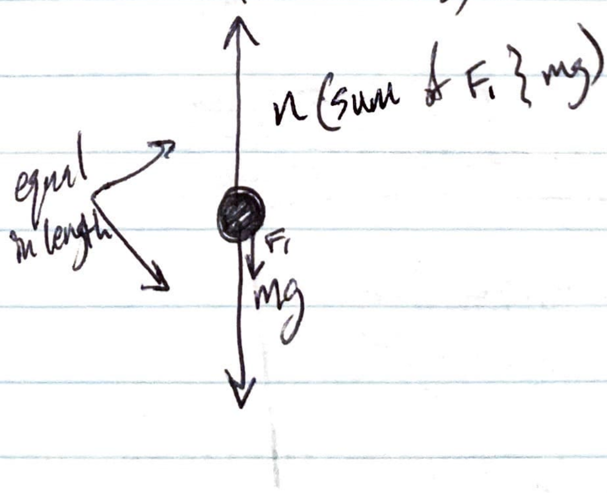
F_1 = contact (push/pull) force

F_T = Tension force $(\vec{T})$



F_{fk} = Kinetic frictional force (f_k)
F_{fs} = static frictional force (f_s)

Box not moving means that $F_1 = F_2$ (Force of box)

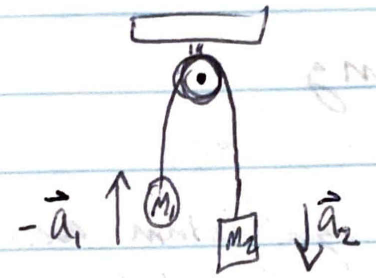


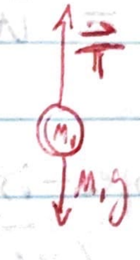
mg = weight of calculator (Earth is pulling calc)

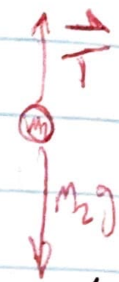
n = norm. force (table is pushing upwards)

F_1 = push down

Ex:







$a_2 = -a_1$

Both came → from $\sum \vec{F} = ma$ where $\sum \vec{F}$ is the net force of tension minus force of gravity

$$M_1 a_1 = (T - M_1 g)$$

$$T = M_1 a_1 + M_1 g$$

$$M_2 a_2 = (M_2 g - T)$$

$$T = M_2 a_2 + M_2 g$$

$$M_1 a_1 + M_1 g = M_2 a_2 + M_2 g$$

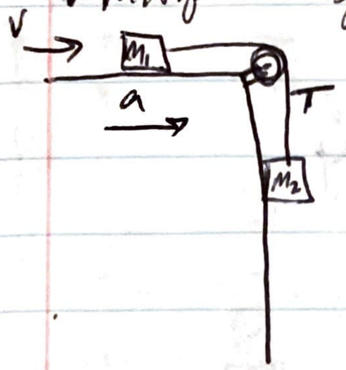
$$M_1 a_1 - M_2 a_2 = M_2 g - M_1 g$$

$$\frac{a_1(m_1 + m_2)}{m_1 + m_2} = \frac{m_2 g - m_1 g}{m_1 + m_2}$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

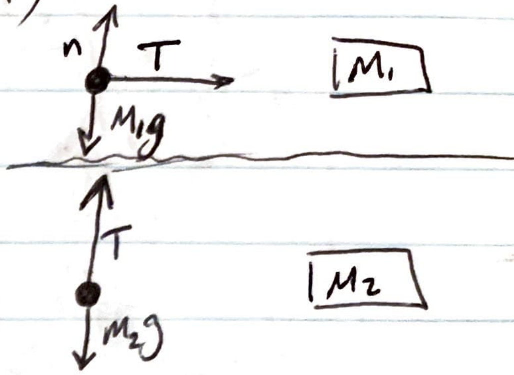
$$\boxed{T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g \qquad a_1 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g}$$

Multiple Object Force Problems (Atwood)



V = same
$M_1 = 5\,Kg$
$\downarrow v \; \downarrow a \quad M_2 = 10\,Kg$

(Find T)



$F_{net_x} = T$ ➡ M_1

$$\boxed{T = M_1 a}$$

← The y-components cancel out since they are =

$F_{net_y} = M_2 g - T$

$$\boxed{M_2 g - T = M_2 a}$$

$$M_2 g - (M_1 a) = M_2 a$$

$$M_2 g = M_2 a + M_1 a$$

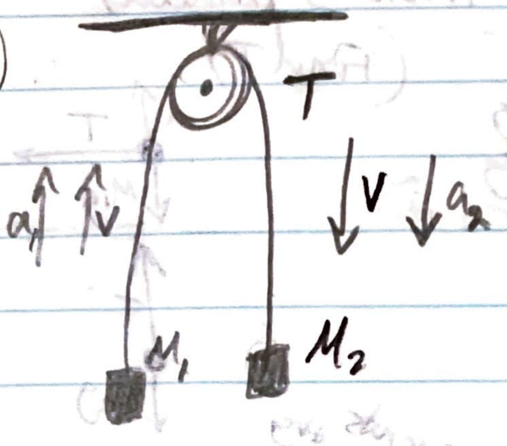
$$M_2 g = a(M_2 + M_1)$$

$$\frac{M_2 g}{M_2 + M_1} = a \quad \rightarrow \quad \frac{10(9.8)}{5+10} = a \qquad a = 6.53$$

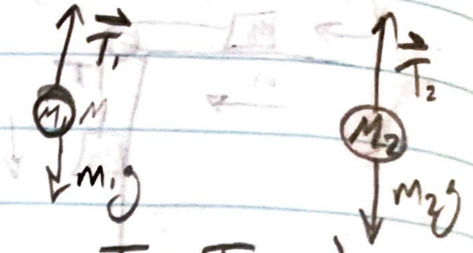
$$T = \frac{M_1 M_2 g}{M_1 + M_2} \qquad T = \frac{5(10(9.8))}{15}$$

$$\boxed{T = 32.7\ N}$$

a.)



b.)



$T_1 = T_2$ } $a_1 = a_2$

$M_1 g$ = gravity exerted on mass 1

T_1 = tension force on mass 1

$M_2 g$ = gravity exerted on mass 2

T_2 = tension force on mass 2

$F_{net} = T - m_1 g \rightarrow \boxed{T - m_1 g = (m_1)a}$

$F_{net} = M_2 g - T \rightarrow \boxed{m_2 g - T = (m_2)a}$

(elimination)

$\rightarrow \cancel{T} - m_1 g = m_1 a$

$+(\cancel{-T} + m_2 g = m_2 a)$

$-m_1 g + m_2 g = m_1 a + m_2 a$ flip

flipped $(m_2 g - m_1 g) = (m_1 + m_2) a$

$$\boxed{a = \frac{(m_2 - m_1)}{(m_1 + m_2)} g}$$

$$a = g \left[\frac{(m_2 - m_1)}{m_1 + m_2}\right]$$

$y = m \qquad x$

** so g can be calculated by finding slope

$a = \frac{T - m_1 g}{m_1}$ $\Bigg\}$ $a = \frac{m_2 g - T}{M_2}$

$\frac{T - m_1 g}{m_1} = \frac{M_2 g - T}{m_2}$

$m_2 (T - m_1 g) = m_1 (M_2 g - T)$

$m_2 T - m_1 g = m_1 M_2 g - T m_1$

$M_2 T + m_1 T = M_1 g + m_1 M_2 g$

$(M_2 + m_1) T = 2 m_1 g + M_2 g$

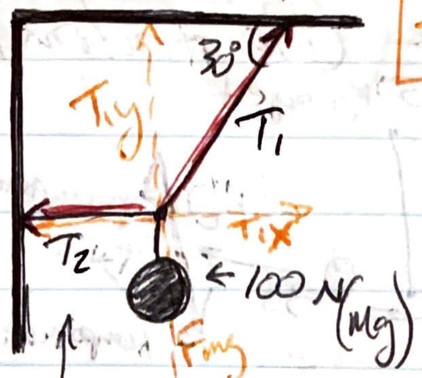
$$\boxed{\vec{T} = \frac{(2 m_1 M_2) g}{(M_2 + m_1)}}$$

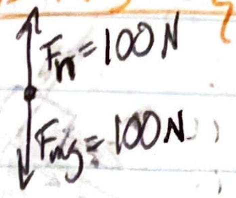
Measure: $t, \Delta y, V_0 (= 0\, m/s)$

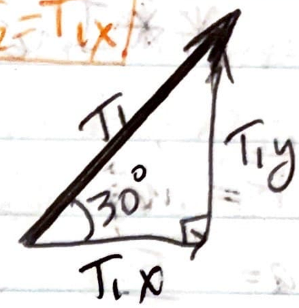
Calculating Tension w/ an

$\boxed{T_{1y} = F_{mg} \;\&\; T_2 = T_{1x}}$

Ex:







system in equilibrium

has no vertical vector component

* The vertical component is equal to the Force exerted by gravity on the object (so $T_{1y} = 100\,N$)

$\sin 30° = \frac{T_{1y}}{T_1}$

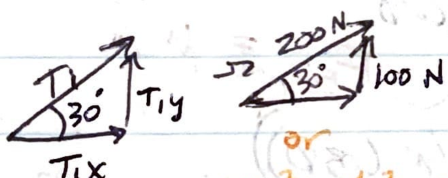
$T_{1y} = T_1(\sin 30°)$

equal to $100\,N = T_1(\sin 30)$

$\frac{100\,N}{\sin 30} = T_1$

$\boxed{T_1 = 200\,N}$

Since $T_2 = T_{1x}$ we could use the trig function from the T_1 vector Δ.



$\tan 30 = \frac{100}{T_2}$

$T_2 = \frac{100}{\tan 30}$

or

$100^2 + V_2^2 = 200^2$

$V_2 = 173.2$

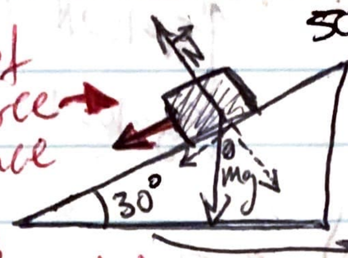
$\boxed{T_2 = 173.2\,N}$

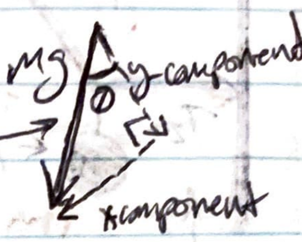
** Inclined Plane (No friction) **

$\Sigma \vec{F} = m\vec{g}$

$a = \dfrac{F}{m}$

Net force since $\vec{N}$ } y-component of mg cancel out





$\varnothing = 30$

$\sin\varnothing = \dfrac{F_{net}}{mg}$

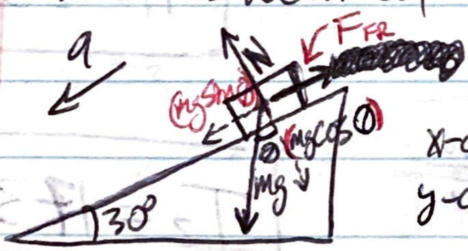
$F_{net} = mg\sin\varnothing$

$a = \dfrac{\cancel{m}g\sin\varnothing}{\cancel{m}} \rightarrow g(\sin\varnothing)$

$= 9.8(\sin 30)$

$\boxed{a = 4.9\ m/s^2}$

** Inclined Plane (W/ Friction) **



$\mu = 0.2$ $F = ma$ mass = 50 kg

$a = \dfrac{F_{net}}{m}$

x-component → $mg\sin\varnothing$

y-component → $mg\cos\varnothing$ = $\vec{N}$

$F_{Friction} = \mu\vec{N}$

$a = \dfrac{F_{net}}{m} = \dfrac{mg\sin\varnothing - (\underbrace{\mu(mg\cos\varnothing)}_{F_{Friction}})}{m}$

$= \dfrac{\cancel{mg}(\sin\varnothing - \cos\varnothing\,\mu)}{\cancel{m}}$

$a = g(\sin\varnothing - \cos\varnothing\,\mu)$

$a = 9.8(\sin 30 - \cos 30)(0.2)$

$\boxed{a = 3.2\ m/s^2}$

Friction Notes

$$\boxed{F_{friction} = \mu N}$$

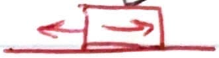
Force of Friction — coefficient of friction — Normal force

μ = coefficient of friction
μ_k = coefficient of kinetic friction
μ_s = coefficient of static friction

μ = Greek letter "Mu"

$0 \leq \mu \leq 1$

nonresponsive force

- <u>Kinetic friction</u> → force needed to keep object going at a constant velocity ($F_{Fr} = \mu_k F_N$) 

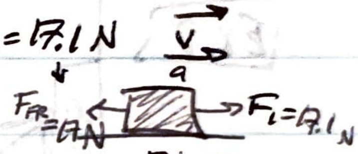
opposition to velocity

- <u>Static Friction</u> → force needed to start motion ($F_{Fr} \leq \mu_s F_N$) → in opposition to applied force / calculated value is <u>max</u>

responsive force

Ex: F_{FR} (static) keeps object from moving

Max: 17 N 

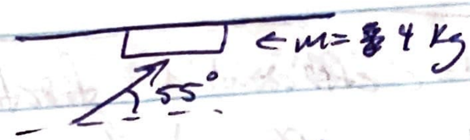
but if $F_1 = 17.1$ N 

$17.1 - 17 = 0.1$ N in the ⊕ direction

$$\boxed{\mu_s = \frac{F_{applied}}{F_{normal}}}$$

** If object is in equilibrium, then "ma" from the $\Sigma F = ma$ equation is 0

Ex.1

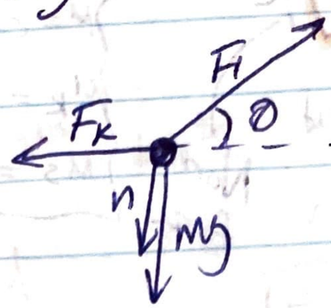


$F_1 = 85\,N @ 55°$

$m = 4\,kg$

$a = 6\ m/s^2$

$\mu_k = ?$



$F_f = \mu_k F_n$

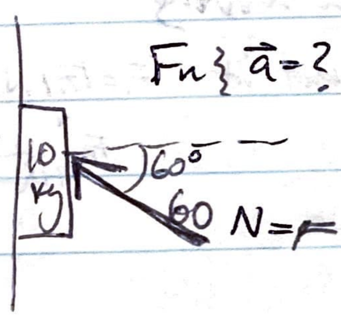
$F_k = \mu_k F_n$

$\mu_k = \frac{f_k}{n}$

$F_{net,x} = F_1 \cos\theta - F_k = ma$

$F_{net,y} = F_1 \sin\theta - mg - n = 0$ ← object is in equilibrium vertically (up/down)

Ex.2



F_n & $\vec{a} = ?$

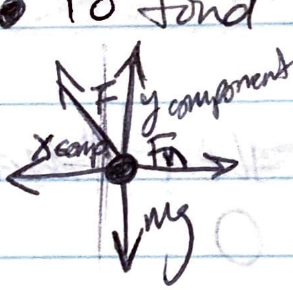
$X = F\cos\theta$

$X = 60(\cos 60)$

$X = 30$ ← Normal Force

$\boxed{F_n = 30\,N}$

• To find $\vec{a}$



$y = F\sin\theta$

$y = 60(\sin 60)$

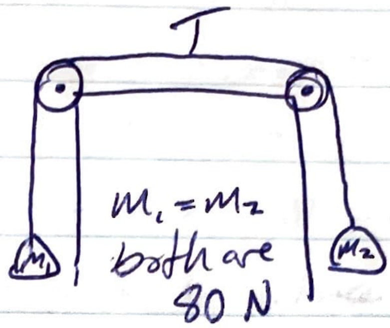
$= 51.96$

$F_{net} = y\text{-comp} - mg = ma$

$51.96 - 98 = 10(a)$

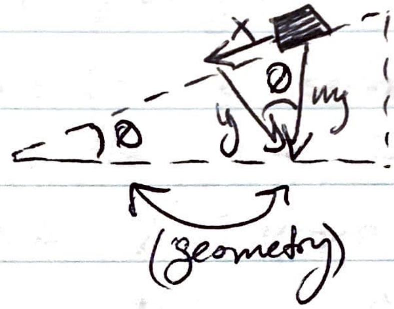
$\boxed{a = -4.6\ m/s^2}$

- Tension



$T = 80\ N$
(not 160 N)

- Vectors on a Ramp



So $x = mg \sin\theta$
$y = mg \cos\theta$

Chapter 5 → Energy

- Work → refers to an activity involving a force & movement in the direction of the force

NOTE: To do work, force must cause displacement (Δx)

* When someone pushes against a wall, then they aren't doing any work since there isn't a Δx

*Work changes energy

W = work

$$\boxed{W = F\Delta x}$$

SI unit: Joule (J)
↳ Newton • meter
or
$Kg \cdot m^2/s^2$

- Work is a scalar quantity

* If the F_1 isn't in the same direction as the displacement then the F_1 vector must be resolved; the X-component or the component that is parallel to Δx is what matters.

$$\boxed{W = (F \cos \theta)\Delta x}$$

X-component ⇒ $F\cos\theta$

- Kinetic Energy (KE) → is the energy the object possesses or stores because of its motion

$$\boxed{KE = \frac{1}{2}mv^2}$$

SI unit: Joule (J)

V = speed
m = mass

Scalar quantity

- Work-Energy Theorem → Net work done by object

$$\boxed{W_{net} = KE_f - KE_i \quad \text{or} \quad W = \Delta KE}$$

* States that the Wnet done by forces on the object causes a change in the KE of it.

Ex:

- Conservative Forces → are forces that store energy such as lifting a book, where the work is against gravity & is available for kinetic energy once it is released Ex: gravity, Elastic forces (Hooke's law)

V_f will give diver KE equal to the work needed to climb against gravity

- Nonconservative Forces → forces that do not store energy; also called dissipative forces; energy that is removed from the system is no longer available to the system for kinetic energy Ex: friction, heat

- Mechanical Energy → the energy acquired by the objects upon which work is done (because when work is done on that object, it gains energy)
  - Ability to do work

$PE = mgy$, $KE = \frac{1}{2}mv^2$ > Sum equals TME

$$\boxed{TME = \sum PE + KE}$$

TME → Total mechanical energy

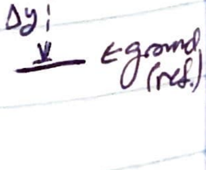
$\sum PE \to PE_s + PE_g$

- Gravitational Potential Energy: $\boxed{PE_{(grav)} = mgy}$ m = mass, g = gravity, $y = \Delta y$
  - is the energy an object possesses because of its position in a gravitational field (or height)

SI: Joule (J)

* Requires 2 objects to [redacted] Form Ex: object & earth

* need a reference such as the ground



- Power → rate at which work is done

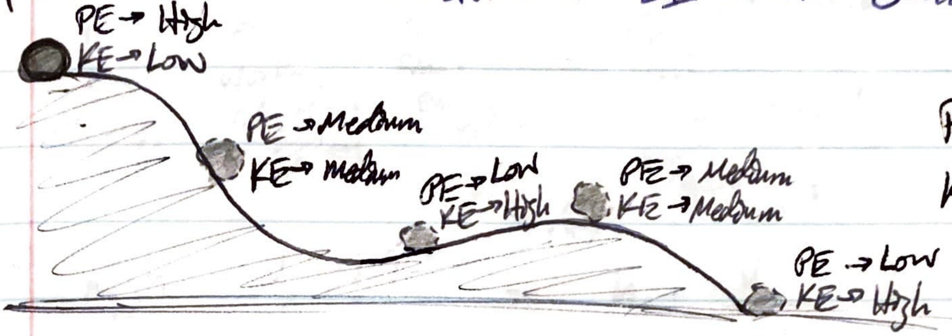
P=power $\boxed{P = W/t}$ SI Unit: Watt

OR

$\frac{\Delta E}{t} = P$

$P = \frac{F\Delta x}{t} \rightarrow P = F\left(\frac{\Delta x}{t}\right) \rightarrow P = F(V)$

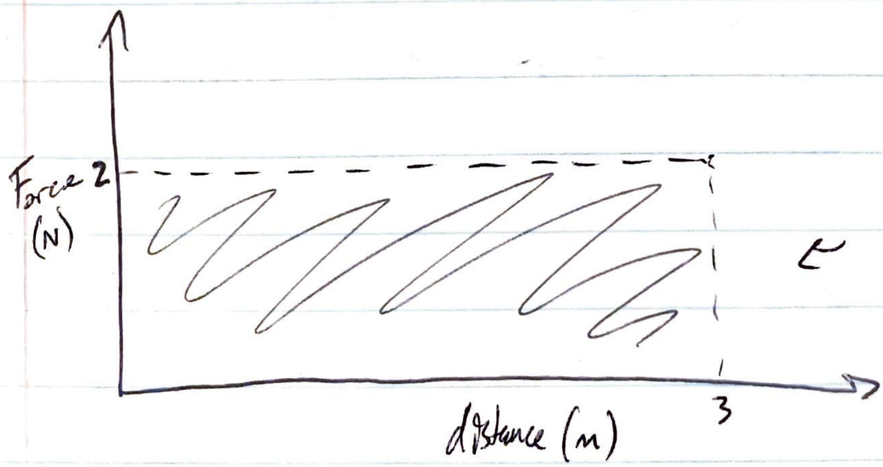
unofficial SI unit: Joules/second



PE → correlates w/ Δy

KE → correlates w/ V

- Hooke's Law $\boxed{F = (K)\Delta x}$ K = spring constant
  - springs have potential energy called Elastic PE
  - springs can exert force (KE)
  - * Compressing a spring → deforming
- Potential Energy of a Spring $\boxed{PE_s = \frac{1}{2}K(\Delta x^2)}$



W = Fd

← The area will determine work

2N(3m) = 6 J

* Friction opposes the motion therefore the work would be ⊖

Conservation of Energy

- Energy is neither created nor destroyed unless acted upon by a net outside force

Ex: 

& $\Sigma F = 0$ So energy is conserved

① $E_0 = E_f$

② $E_0 + W = E_f$ → ③ $\cancel{KE_0} + PE_{g0} = KE_f + \cancel{PE_{gf}}$

E_0 ← initial energy; W ← work added to system; E_f ← final energy

$\cancel{KE_0}$ ← object isn't moving initially; PE_{g0} ← ΣPE of gravity & spring

④ $mgh_0 = \frac{1}{2}mV_f^2$ → ⑤ $h_0 = \frac{V_f^2}{2g}$

V_f ← final velocity

Example of Steps to calculate energy problems

- Steps to Calculate energy Problems
  1.) Determine if ME is conserved or not — $ME_0 = ME_f$
  2.) Expand mechanical Energy types — $ME_0 + W = ME_f$
  3.) Eliminate zero/irrelevent terms — $\cancel{KE_0} + PE_0 + W = ME_f \ldots$ (0)
  4.) Givens/unknowns
  5.) Expand to include equations (mgh, $\frac{1}{2}mv^2$, $\frac{1}{2}Kx^2$)

Power Continued

- Unit of Power in US customary system is in HP (Horsepower) $1 hp = 550 \frac{ft.lb.}{s} = 746$ Watts

** Maximum Power Output from Humans over various Periods

Power	time
2 hp / 1500 watts	6 s
1 hp / 750 Watts	1 min
$\approx$ 0.35 hp / 260 watts	1/2 hr
0.1 hp / 75 watts	8 hr

- KWh → Kilowatt-hour

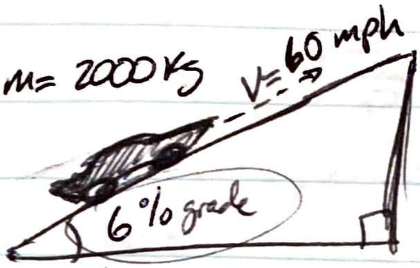
1 KWh is the energy transfered in 1 hour at the constant rate of 1 KW = 1000 J/s

$$1\ KWh = (10^3 W)(3600 s) = (10^3 J/s)(3600 s) = 3.60 \times 10^6\ J$$

* KWh is a unit of energy NOT POWER

- Center of Mass (CM) → point in the body at which all the mass may be considered concentrated

Car up a Hill



$$\frac{1\ \text{mph}}{0.44704\ \text{m/s}} = \frac{60\ \text{mph}}{?\ \text{m/s}}$$

26.8 m/s

Find $P = ?$

Conversion Factor for mph to m/s:

$$\boxed{\frac{1\ \text{mile}}{\text{hr}} = 0.44704\ \text{m/s}}$$

$\frac{6}{100}$

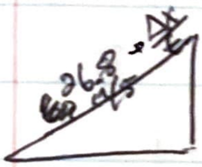
arctan

$\tan^{-1}\left(\frac{6}{100}\right) = 3.434^\circ$

$P = \frac{W}{t}$ ← since the car is traveling @ constant $\vec{v}$, the $\vec{a}$ is $0\ \text{m/s}^2$ thus making the work $= 0\text{ J}$. But the car is exerting a change in force (PE_g) so $P = \frac{\Delta E}{t} = \frac{\Delta mgy}{t}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\text{opp} = \text{hyp}(\sin\theta)$



$$\frac{\Delta y}{t} = \frac{\Delta x}{t}(\sin\theta)$$

$$\boxed{\frac{\Delta y}{t} = v(\sin\theta)}$$

$$P = mg\boxed{\left(\frac{\Delta y}{t}\right)}$$

$$P = mg\left(\frac{\Delta y}{t}\right)$$

$$P = mg\left(v(\sin\theta)\right)$$

$m = 2000\ \text{kg}$

$g = 9.8\ \text{m/s}^2$

$v = 26.8\ \text{m/s}$

$\theta = 3.434^\circ$

$$P = \left(2000(9.8)\right)\left((26.8)(\sin 3.43)\right)$$

$$\boxed{P = 31{,}427\ \text{Watts}}$$

To convert Watts to Hp

$$\boxed{\frac{W}{746} = \text{Hp}}$$

$$\frac{31{,}427}{746} = \boxed{42\ \text{Hp}}$$

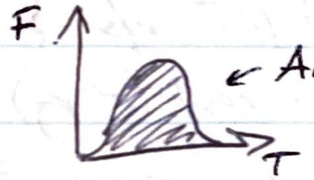
↑ need to convert because the initial velocity given was not in SI

Chapter 6 - Momentum / collisions

- Momentum $\boxed{\vec{p} = m\vec{v}}$ $\vec{p}$ → linear momentum
  SI: Kilogram-meter per second ($kg \cdot m/s$)
  - a vector quantity
  - when no ΣF (external) acts on system the $\Sigma \vec{p}$ of the system remains constant in t
  - Has inertia

$\Delta p = F \Delta t$ * In order to change momentum a force over a time period must be applied (AKA Impulse)

- Impulse $\boxed{\vec{I} = \vec{F}(\Delta t)}$ SI: $K \cdot \frac{m}{s}$



← Area = Impulse

~~[illegible]~~ Impulse Changes Momentum

** The law of Conservation of Momentum
↳ The total momentum of all objects interacting w/ one another will remain constant regardless of the Nature of Forces between the objects

$$\boxed{\Sigma p_0 = \Sigma p_f}$$

↳ $\vec{I} = \vec{F}\Delta t$ ➡ $\vec{I} = \Delta\vec{p}$ ➡ $m\vec{v}_f - m\vec{v}_0 = \vec{I}$

- Collisions

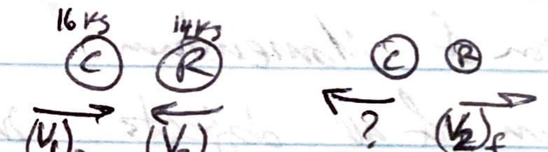
$\vec{P}$ is conserved

- Elastic → both $\vec{p}$ & KE are conserved
- Inelastic → $\vec{P}$ is conserved; KE is transferred to sound/heat & deformation
- Perfectly ~~inelastic~~ → $\vec{p}$ is conserved, KE is not & the 2 objects stick together after collision so their V_f are same ← one object is deformed

$$\boxed{V_f = \frac{(m_1 V_1)_0 + (m_2 V_2)_0}{m_1 + m_2}}$$

- Explosion → ~~[illegible]~~ KE isn't conserved; start with one object & breaks into other parts; $\vec{P}$ is conserved; Ex: recoil

$\boxed{\text{EX 1}}$ 16 kg canoe moving to the right 12.5 m/s & makes an elastic head-on collision w/ a raft (14 kg) moving to the left 16 m/s. After collision, raft moves 14.4 m/s



$\boxed{\text{Right } \oplus \quad \text{Left } \ominus}$

$$P_0 = P_f$$

$$(P_1)_0 + (P_2)_0 = (P_1)_f + (P_2)_f$$

$$(m_1 V_1)_0 + (m_2 V_2)_0 = (m_1 V_1)_f + (m_2 V_2)_f$$

$$V_{1f} = \frac{(m_1 V_1)_0 - (m_2 V_2)_f}{m_1}$$

$m_1 = 16$ kg

$m_2 = 14$ kg

$(V_1)_0 = +12.5$ m/s

$(V_2)_0 = -16$ m/s

$(V_1)_f = -?$

$(V_2)_f = +14.4$ m/s

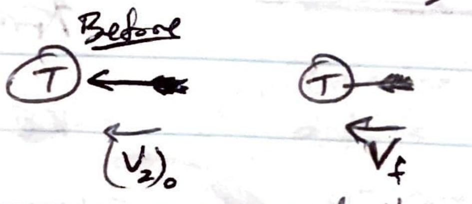
$$(V_1)_f = \frac{(16(12.5)) - (14(14.4))}{16}$$

$$\boxed{(V_1)_f = 14.1 \text{ m/s left}}$$

$$\boxed{KE = 3040 J}$$

Ex.2 0.25 kg arrow w/ $\vec{v} = 12\ m/s$ to the west pierces center of a 6.8 kg target

perfectly inelastic



$$V_f = \frac{(m_1 v_1)_0 + (m_2 v_2)_0}{m_1 + m_2}$$

$(P_2)_0 = (P_{sys})_f$

combined V_f

✓ Perfectly inelastic

$KE_0 \neq KE_f$

$$\boxed{V_f = 0.43\ m/s}$$

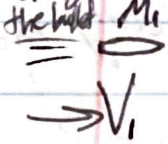
$$\boxed{KE = \approx 17J}$$

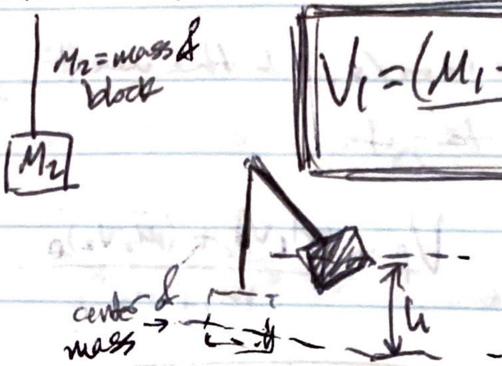
Ex.3 63 kg astronaut's tether line breaks he throws a 10 kg O_2 tank w/ $\vec{v}$ of 12 m/s what is V_f?

Ballistic Pendullum

$M_1 \Rightarrow$ mass 1

$V_1 \Rightarrow$ velocity of the bullet





$$\boxed{V_1 = \frac{(M_1 + M_2)V_2}{M_1}}$$

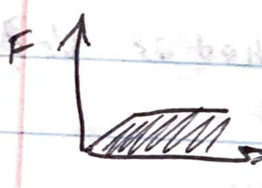
$$\boxed{V_2 = \sqrt{2gh}}$$

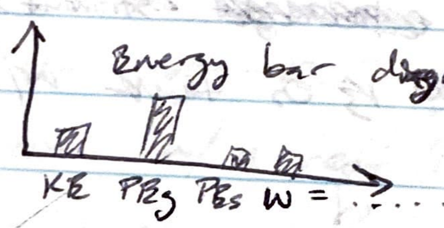
↑ From conserv. of energy eqn.

$KE = PE$

$\frac{1}{2}(m_1+m_2)v^2 = (m_1+m_2)gh$

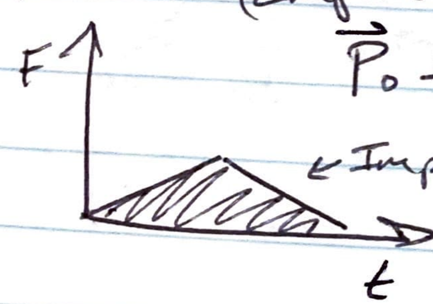
- F vs Δx (Work)

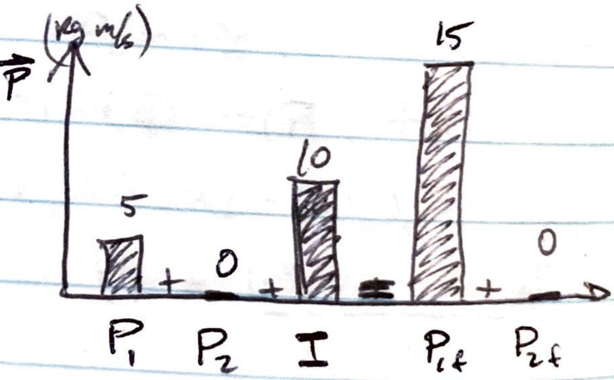


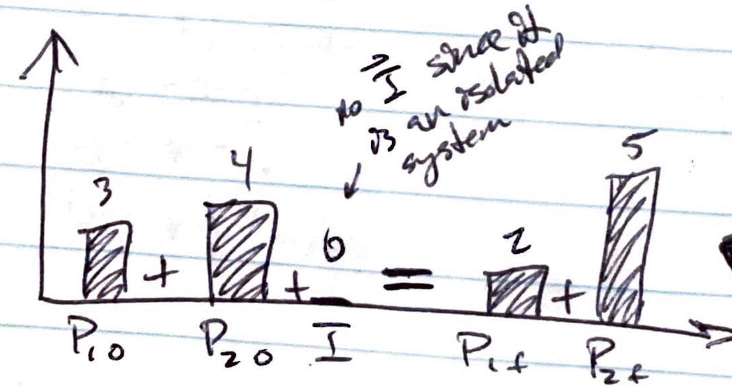


- F vs t (Impulse)

$$\vec{P}_0 + I = \vec{P}_f$$





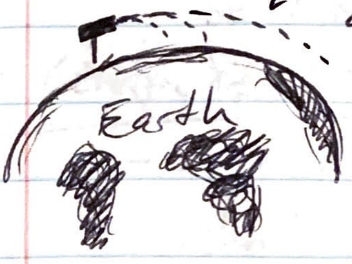


← collision

Newton's Law of Universal Gravitation (NLUG)

- Things in space orbit in free fall because the only influence is gravitational forces

If launched at a very high $\vec{v}$ then the objects free fall would not beat the curvature of the (E)



- Centripetal force (holds the planets in orbit → gravit. F)
- Gravit. force is a mutual force of attraction
  - ↳ depends on the masses & the distance btwn them

$F \propto (M_1 M_2)/r^2$ ← r = center of mass to center of mass distance

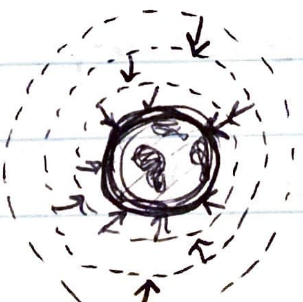
↑ directly proportional

← Ex

$F = G\left(\frac{M_1 M_2}{r^2}\right)$ OR $\boxed{F_{gravity} = \frac{G m_1 m_2}{r^2}}$

$G = 6.673 \times 10^{-11}\ N \cdot \frac{m^2}{kg^2}$

- G → constant of universal gravitation
- Gravity is a field force / pulling force
- The r^2 in the formula comes from the SA of a sphere $SA_s = 4\pi r^2$



$\boxed{\text{Ex. 1}}$ Astronaut standing on asteroid Ceres (radius of 5.1 x10² km) drops a rock from a height of 10 m. It takes 8.06 sec to hit ground.

$\Delta y = \frac{1}{2} a t^2$

$10 = \frac{1}{2}(a)(8.06^2)$

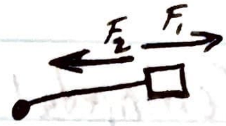
$a = 0.31\ m/s^2$

a) $\boxed{0.31\ m/s^2 = a}$

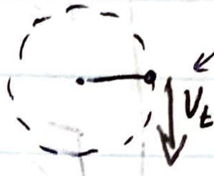
F=

Circular Motion

- centripetal $\vec{F}$ is a normal force



- Tangential speed (V_t)



- Centripetal acceleration
  - as it moves in a ⊙ path, its direction changes which means it is accelerating

$$a = \frac{\Delta V}{t}$$

← Center seeking

** depends on V_t } radius

- ($\vec{a}_t$) → tangential acceleration
  - where there is an $\vec{a}$, there must have a $\sum \vec{F}$

** Inertia keeps the object from coming onto the center

- Types of Forces that cause circular motion:
  - Tension (ex: rope, cable)
  - Normal force
  - Friction
  - gravity (orbit) / Magnetic

$$\boxed{a_c = \frac{V_t^2}{r}}$$

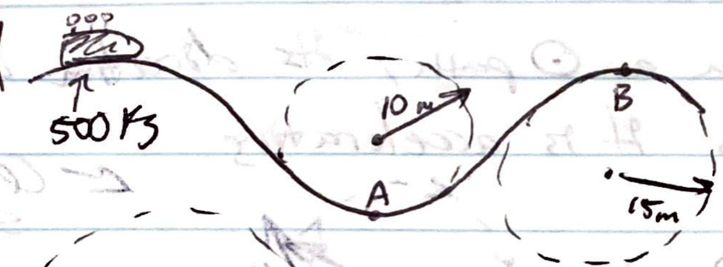
a_c = acceleration centripetal
V_t = Velocity tangental
r = radius

↑ POINTS TO CENTER

- Centripetal Forces are $\sum$ Forces

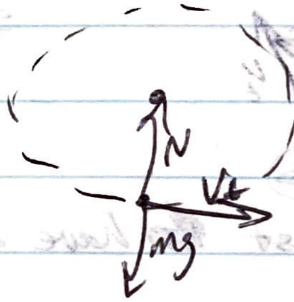
$$\boxed{\begin{gathered} F_c = ma_c \\ \boxed{\text{or}} \\ F_c = m\left(V_t^2/r\right) \end{gathered}}$$

Ex. 1



① If @ pt A = 20 m/s What is Force of the track on cart (AKA Normal force)

$\Rightarrow \boxed{24{,}900 \text{ N}}$



$$\sum F = n - mg = ma_c$$

$$n = \frac{mv^2}{r} + mg$$

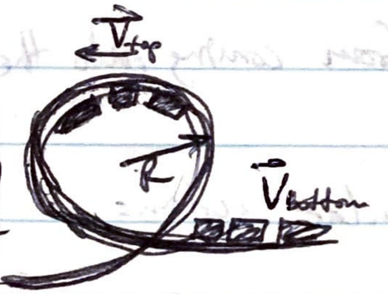
② What is max $\vec{V}$ the vehicle can have @ pt B for gravity to hold it on track

$$mg - \cancel{N} = \frac{mv^2}{r}$$

$$4900 = \frac{500(v)^2}{15}$$

$$\boxed{v = 12 \text{ m/s}}$$

Ex. 2



① What $\vec{V}$ must it have to just make it over the track? (w/o assistance from track)

↳ N = 0 N

$$\boxed{V = \sqrt{Rg} \text{ m/s}}$$

Fnetc
↓
$\sum F$ @g
⊙

$$F_{net} = ma_c$$

$$mg + \cancel{N} = ma$$

$$\cancel{m}g + \cancel{N} = \cancel{m}\left(\frac{V_t^2}{R}\right)$$

$$V_t = \sqrt{R(\cancel{m}g)}$$

② What $\vec{V}$ will it have @ the bottom?

Top = Bottom

$$KE_{top} + PE_g = KE_{Bot}$$

$h = 2R \rightarrow \frac{1}{2} m V_{top}^2 + mgh = \frac{1}{2} m V_{bot}^2$

(AKA diameter)

$$2 \cdot \left(\frac{1}{2}\cancel{m}(Rg) + \cancel{m}g(2R)\right) = \left(\frac{1}{2}\cancel{m}V^2\right) \cdot 2$$

$$\boxed{V = \sqrt{5Rg} \text{ m/s}} \Longleftarrow \cancel{m}(Rg + 4R) = \cancel{m}V^2$$

③ What is N @ bottom w/ r = 10.0m

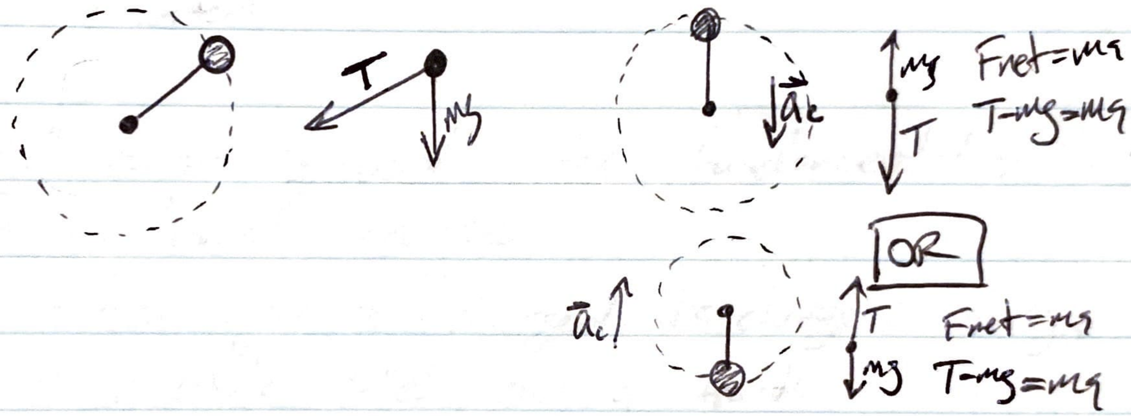
$n = \frac{mv^2}{r} + mg$ ← From equation in #1

$n = \frac{m(R5g)}{R} + mg$ $\boxed{N = 6mg}$

Notes Cont....

- $\vec{a}_c$ points to center of circle
- Set Fnets based on forces present (points to center)

Ex:



Chapter 15 → Electricity

- Electrical Charges
  - Protons (+) / Electrons (−) / Neutrons (neutral)
  - LIKE charges repel one another & UNLIKE charges attract one another

* How electrons & protons differ
  - Electrons has the same magnitude charge as protons but w/ opposite signs
  - Electrons are far lighter than protons thus $\vec{a}$ is greater

amount of charge per electron

- "e" is the symbol of the fundamental unit of electrical charge
  - ↳ If an object is charged, its charge is always a multiple of "e" never a fraction

    Ex: ±10e or ±3e <u>not</u> ~~±0.5e~~
  - ↳ The charge is said to be "Quantized" meaning that it is in discrete chunks & <u>can't</u> be subdivided

    ** conducted by Robert Millikan (scientist) in 1909
- The value of e is known to be $\boxed{1.60219 * 10^{-19}\ C}$

  (SI unit of electric charge is the Coulomb or C)

(+1) (+1)

adds a charge

↓

(+0) (+2)

- Electric Charge is <u>always</u> CONSERVED
  - ↳ charge isn't created, rather both objects are charged because ⊖ charge is transferred from 1 to another

- Coulomb's Law (used to find F_e btwn stationary particles)
  - Electric forces have these properties:
    1. It is directed along a line joining the 2 particles & is inversely proportional to the separation distance "r" between them.
    2. It is proportional to the product of the magnitudes of the charges, $|q_1|$ & $|q_2|$ of the 2 particles
    3. It is attractive if the charges are of opposite signs & repulsive if the charges have the same sign

$$\boxed{F = K_e\left(\frac{|q_1| * |q_2|}{r^2}\right)}$$

K_e = Coulomb constant

F = "electrostatic force"

($K_e = 8.9875 * 10^9 \ N \cdot m^2/C^2$)

**(r is in meters)

- Com & Con Gravitational/electrical forces
  - Both act at a distance (r)
  - Both are inversely proportional to distance squared (r^2)
  - $F_G = G\frac{m_1 m_2}{r^2}$ & $F_e = K_e\frac{|q_1||q_2|}{r^2}$ ← similar formulas
  - Differ in that "F_e" can be attractive or repulsive & F_g → always attractive
  - F_e is always stronger between the same particles as F_G

- Factors affecting the force strength/direction
  - "r" (distance between objects/particles)
  - The magnitude of the charges of both particles

- Electric current is the flow of electric charge
  - ↳ loses an electron → substance becomes (+) charged
  - ↳ gains an electron → substance becomes (−) charged

- Grounding → electrically neutralizes the current

** The Flow of electricity (rate at which charge passes through a given area)

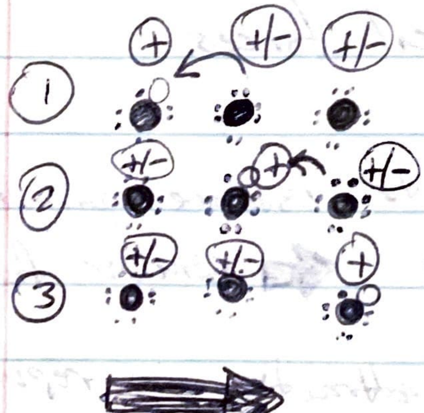
$$\boxed{I = \frac{\Delta Q}{\Delta t}}$$

I = current

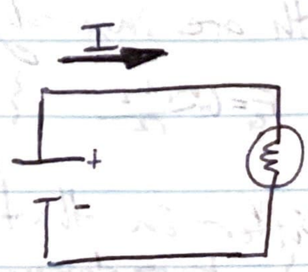
ΔQ = change in the charge

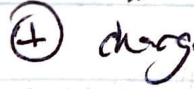
SI unit: $\frac{\text{Coulombs}}{\text{sec}}$ = Amperes OR "Amps" OR "A"

- current carries energy throughout a circuit



current flows in the direction of (+) charge

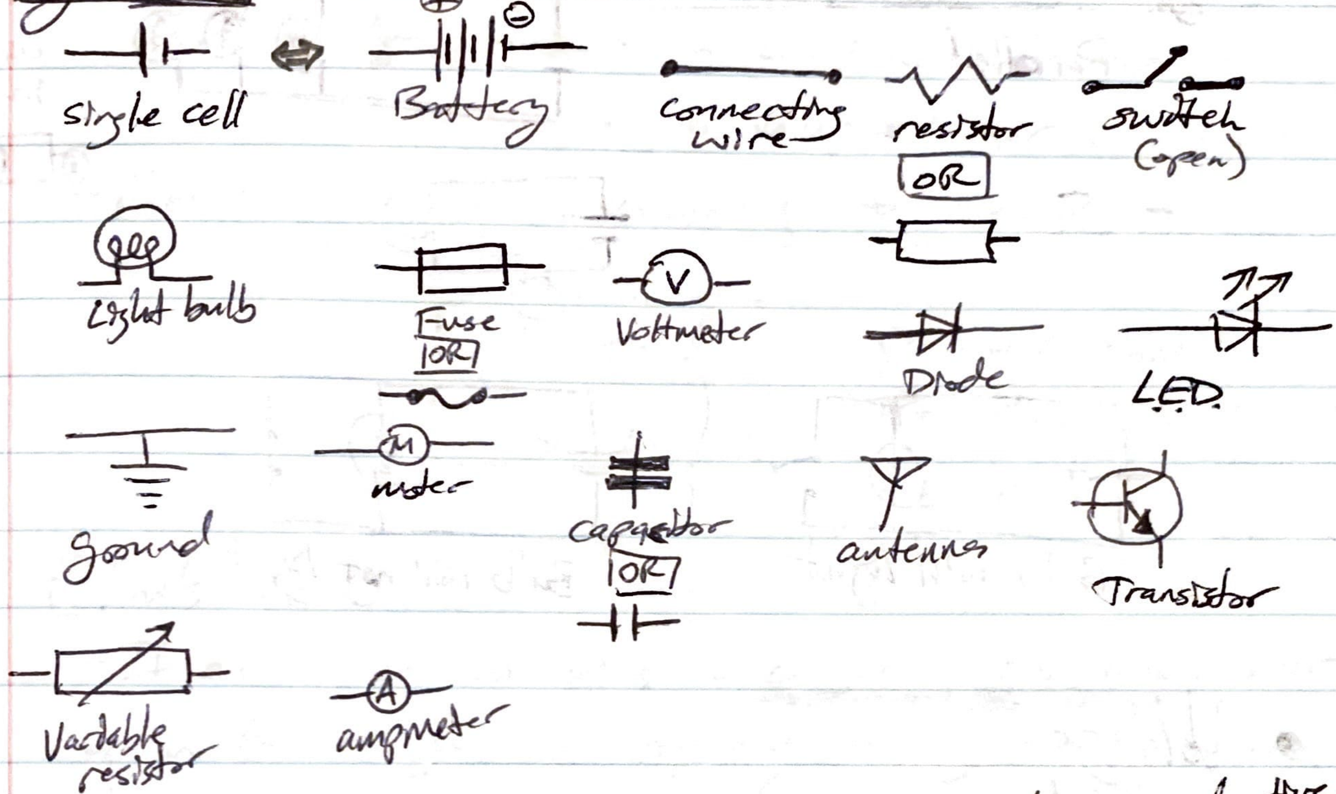




(+) charge moves this way

Circuits

- **Symbols**

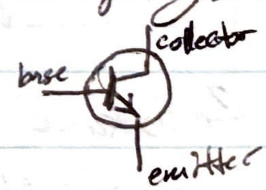


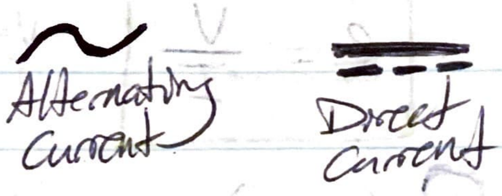
- **Capacitor**

  - an electronic component used to store energy

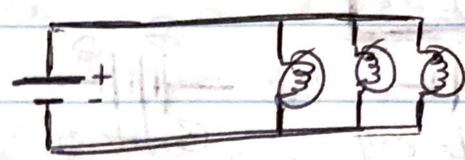
- **Transistor**

  - a semiconductor device used to amplify/switch electronic signals & electronic power

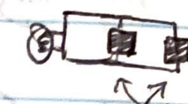




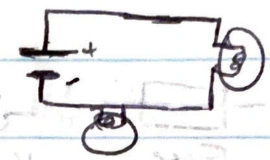
- **Types of Circuits**
  - Parallel

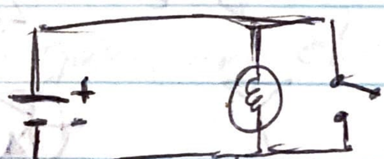


** Batteries can also be in parallel

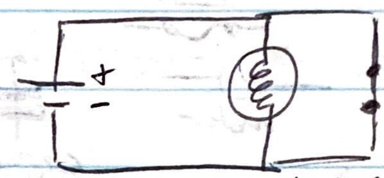


  - Series





Bulb will light



Bulb will not light

- **Voltage**
  - determines energy per charge
  - SI unit: $\frac{\text{Joules}}{\text{Coulomb}} \Rightarrow$ Volt Abv: "V"
  - AKA "Electric Potential" or "Potential Difference" or "emf" (electro-motive force) → $\boxed{\mathcal{E}}$
    ↓ Emf
- **Resistance**
  - opposition to current

$$\boxed{R = \frac{V}{I}}$$

R = resistance
V = voltage
I = current

SI unit: Ohm or Ω (omega)

  - depends on the shape, temp, & composition of an object

$\boxed{R \text{ proportional to } l}$

l = conductor (length of)

$R \propto l$ longer $\rightarrow$ greater resistance

$R \propto \frac{1}{A}$ wider cross sectional area $\rightarrow$ smaller resistance

higher temp $\rightarrow$ larger resistance

- Resistivity (ρ) : greater ρ $\rightarrow$ greater R
  row

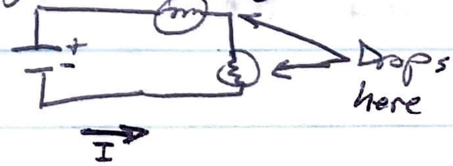
$$\boxed{R = \frac{\rho l}{A}}$$

ρ = row (constant)
l = length of conductor / conductivity
A = area

- Ohm's Law
  - If an object maintains a constant resistance over a wide range of applied voltages or currents then it obeys Ohm's law (it is "ohmic")
  - The current that flows in a circuit is proportional to voltage & inversely prop. to resistance of circuit
  - Both come from R equation $\boxed{I = \frac{V}{R}}$ OR $\boxed{V = IR}$

  - Voltage Drop $\rightarrow$ amount of energy per charge that is delivered in the system

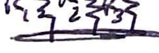
$\Delta V \rightarrow$



* In a Series circuit:

$$R_T = R_1 + R_2 + R_3 + \ldots$$

* In a Parallel circuit



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Ex:1 An electric heater is operated by applying a potential difference of 50 V across a wire of a total resistance of 8.00 Ω

a) Find current

$I = V/R$

$I = 50/8 \rightarrow \boxed{6.25 \text{ Amps}}$

Ex. 2 • Derive expression for Power in terms of R & V

$P = \frac{\Delta E}{t} \rightarrow P = V \cdot I \rightarrow \boxed{P = \frac{V^2}{R}}$

$I = \frac{V}{R}$

• Derive expression for Power in terms of R & I

$P = V \cdot I \qquad V = IR$

$\boxed{P = I^2 R}$

Electrical Power { Energy

- power → rate @ which energy is being used

$P = \frac{\Delta E}{t}$ * Electric power is the rate @ which electrical energy is changing

* Power determines brightness

- The amount of energy depends on:
  1.) current flowing (moving charge) $I = \frac{Q}{t}\left(\frac{\text{coulombs}}{\text{sec}}\right)$
  2.) The amount of energy per charge (voltage) → energy in back pack

$V = \frac{E}{Q}\left(\frac{\text{Joules}}{\text{Coulomb}}\right)$

SI unit: Watt (W) $\boxed{P = V \cdot I}$

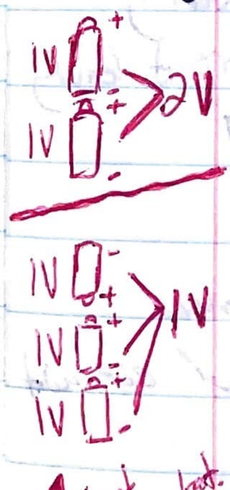
P = electrical power
V = voltage
I = current

↳ $\frac{E}{t} = VI \rightarrow \frac{E}{t} = P$

$V = IR$ $P = I^2R$

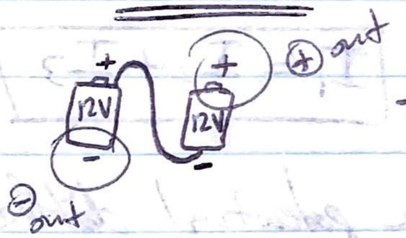
$E = P \cdot t$ SI: Watt-sec

- Batteries



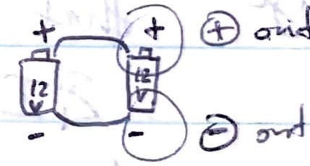
Subtract 1V since bat. is facing other way

In Series

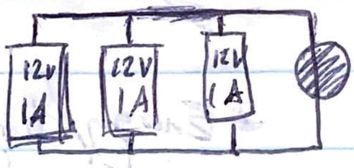


⇒ 24 Volts total
mA is constant

In Parallel



⇒ 12 Volts total
mA increases



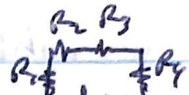
12V
3a
36 W

- Continuity refers to being part of a complete/connected whole. In electronic applications, when an electric circuit is capable of conducting current it demonstrated continuity. (0 Ω) ← no resistance

- **Series Circuits (cont...)**

R_{in} a circuit

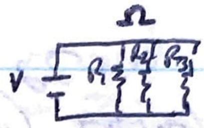
* Resistors in a series



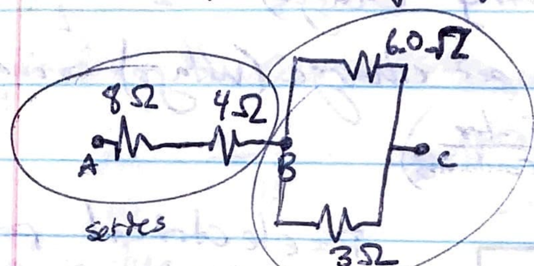
$$\boxed{R_T = R_1 + R_2 + R_3 + R_4 \ldots}$$

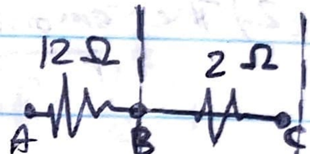
- **Parallel Circuits**

* resistors in parallel



$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$





$8\Omega + 4\Omega$

$R = 12\Omega$

$\frac{1}{R} = \frac{1}{6} + \frac{1}{3}$

$\frac{1}{R} = \frac{1}{6} + \frac{2}{3}$

$\frac{1}{R} = \frac{3}{6} = \frac{1}{2}$

$R = 2\Omega$

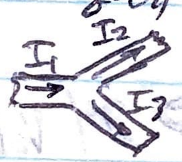
Total 14Ω

steps →
1.) Add reciprocals of resistors
2.) Flip final answer

- **Kirchhoff's Junction/Loop Rules**
  - Junction Rule: The sum of the currents entering any junctions must equal the sum of the current leaving that junction

Conservation of Charge

$\Sigma I = 0$



$$\boxed{I_1 = I_2 + I_3}$$

  - Loop Rule: The sum of the potential differences across all the elements around any closed circuit loop must be zero

Conservation of energy

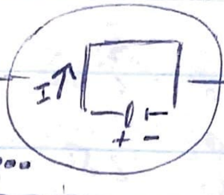
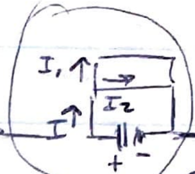
* Energy/charge is conserved

Junction Rule

$$\boxed{\Sigma I_{in} = \Sigma I_{out}}$$

$$+I_1 - I_2 - I_3 = 0$$

Electrical Conservation

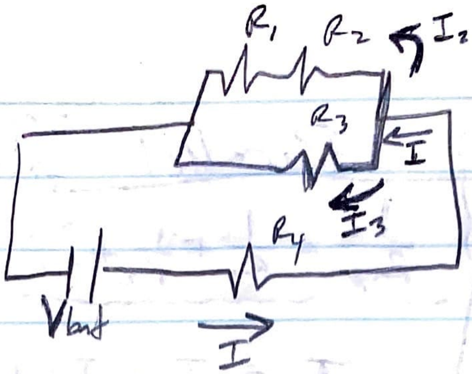
	Series 	Parallel 
Resistance (equivalent)	$R_{eq} = R_1 + R_2 + R_3 + \dots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$ $\boxed{OR}$ $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$
Current	$I_0 = I_1 = I_2 = \dots$	$I_0 = I_1 + I_2 + I_3 + \dots$
Voltage	$V_{battery} = \underbrace{V_1 + V_2 + V_3 + \dots}_{\text{Voltage drops}}$	$V_{battery} = V_1 = V_2 = V_3 = \dots$

- **Loop Rule** (cont...)

$$+V_{bat} - V_1 - V_2 = 0$$

$$+V_{bat} - I_1 R_1 - I_2 R_2 = 0$$

EX



$R_1 = 8\,\Omega$
$R_2 = 2\,\Omega$
$R_3 = 10\,\Omega$
$R_4 = 12\,\Omega$
$V_{bat} = 40\text{ V}$

a.) R_{eq}

$\left(\frac{1}{10}+\frac{1}{10}\right)^{-1} + 12\,\Omega$

$5 + 12 = 17\,\Omega$

$\boxed{R_{eq} = 17\,\Omega}$

R_4

*Current in a parallel config. has the same current Amperage

$I = \frac{40}{12} = 2.35\text{ A}$

Voltage Drop $= 28.2\text{ V}$

R_3

$V = 11.8\text{ V}$
$(40 - 28.2)$

$I = \frac{V}{R}$ $\quad I = \frac{11.8}{10}$

$I = 1.18\text{ A}$

R_1

$I = \frac{11.8}{8}$

$I = 1.175$

R_2

$I = \frac{11.8}{2}$

Battery

$I = \frac{V}{R}$

$I = \frac{40}{17}$ ← Total res.

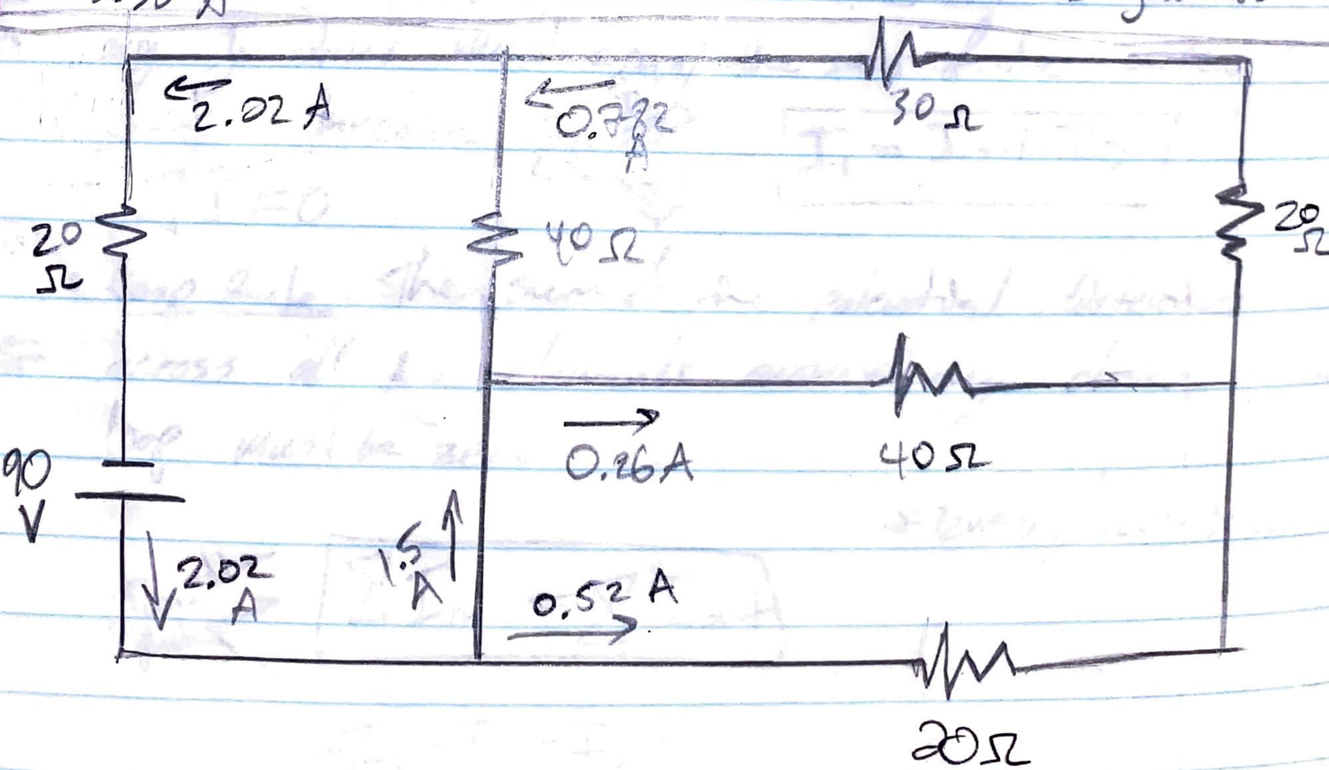
$I = 2.35\text{ A}$

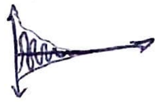
$P = IV$

$P = (2.35)(40)$

$\boxed{P = 94\text{W}}$

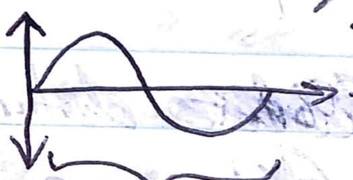
* If these resistors were lightbulbs, then "R_4" would be the brightest.





Chapter 13 - Simple Harmonic Motion

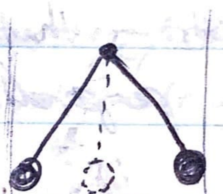
- S.H.M. → comes from harmony (multitudes of cycles repeated over and over again) AKA periodic motion
  - a system in harmonic motion is called an oscillator
  - Equilibrium is maintained by restoring forces (is any force that acts to pull the system back towards equilibrium) Ex: gravity



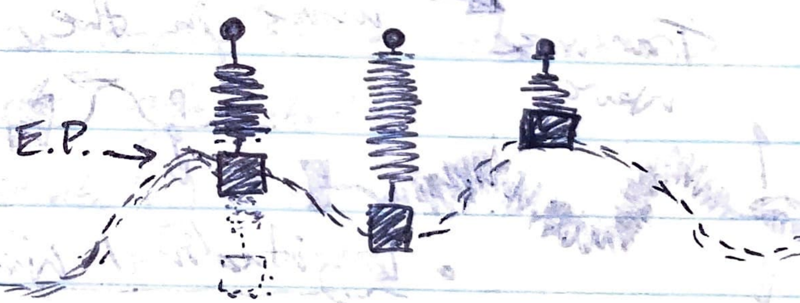
} A or amplitude (max amount system moves away from equil. position)

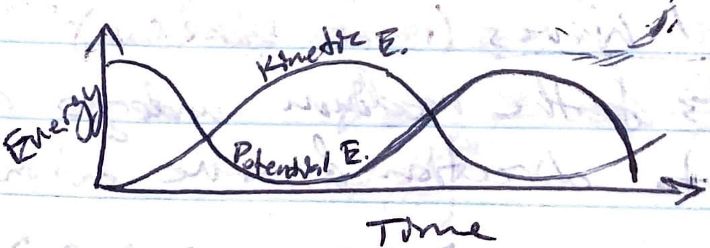
Period or P

- Frequency → # of cycles per Δt (1 second)
  - ↳ SI units: Hertz (Hz)
- Period → ~~time~~ amount of time to complete 1 full cycle 



E.P.





← Energy vs. Time for Harmonic Motion

- Hooke's Law $\boxed{F_s = -Kx}$ x ⇒ displacement from equilibrium position

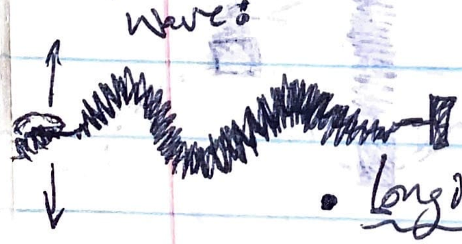
  * $-K$ because the Force exerted by the Spring is always directed opposite the displacement of the object.

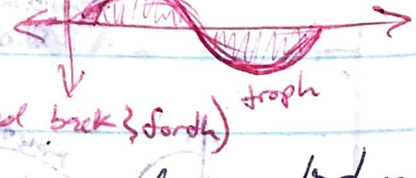
- Period (T_s) $\boxed{T_s = 2\pi\sqrt{\frac{m}{K}}}$

- Waves (mechanical)
  - considered motions of disturbances that travel through a medium (AKA a propagation of disturbances)
    ↳ need it unlike electromagnetic waves
  - deviations from a state of rest
  - Types of waves
    - Traveling wave

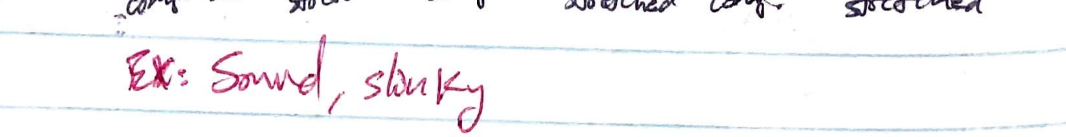
      - each segment of the string that is disturbed moves in the direction perpendicular to wave's motion

Transverse wave:





- Longitudinal Waves (Pumped back & forth)
  - elements of the medium undergo displacement parallel to direction of wave motion



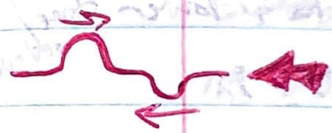
Ex: Sound, slinky

- Wave Superposition
  - when 2 or more waves encounter each other while moving through medium; the resultant wave is found by adding together the Δx of the individual waves point by point

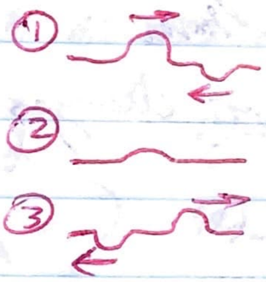
Superposition Principle

- Types:

  - * Constructive interference
    - meet Crest to Crest [OR] Trough to Trough
  - * Destructive interference



    - meets Crest to Trough or vice versa



NOTE: 2 Traveling waves can meet/pass through each other without being destroyed or even altered

- Frequency/Amplitude/Wavelength
  - Amplitude → max Δx string travels above/below point of equilibrium
    (A)
  - Wavelength λ (lambda) → Δx between 2 pts.
  - Velocity of a wave

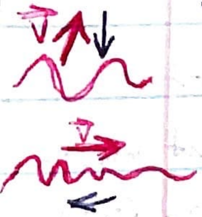
$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = f\lambda$$

Period (T)

$$\boxed{v = f\lambda} \quad \text{or} \quad \lambda = \frac{v}{f}$$

$E \propto A^2$

- The rate at which a wave transfers energy depends on A.

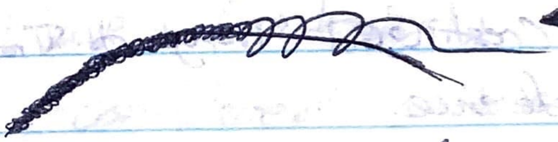


- 2 speeds:
  - speed of physical string that vibrates up/down
  - wave speed → rate at which disturbance moves/shifts in x direction

$$\boxed{\vec{V}=\sqrt{\frac{F}{\mu}}}$$

F = tension in the string in N
μ = mass of string per unit length (AKA linear density)

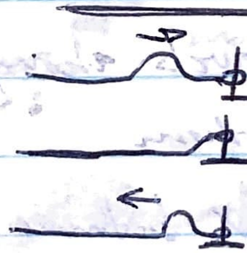
EX: winding on a bass guitar string



← adds thickness thus more mass per length (μ) thus lower wave $\vec{v}$ & frequency resulting in a lower tone/octave

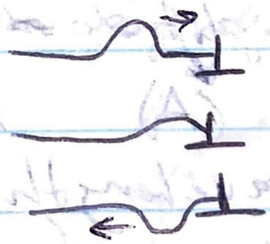
- The speed of a mechanical wave is constant for a given medium
  - a change in the medium changes speed

- Wave Reflection
  - Free Boundary



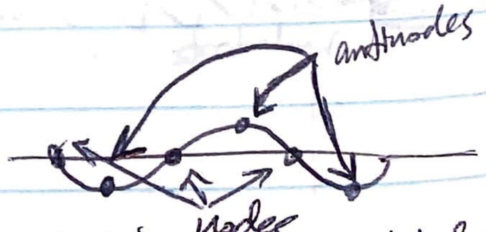
*Loop at the end of string

  - Fixed Boundary



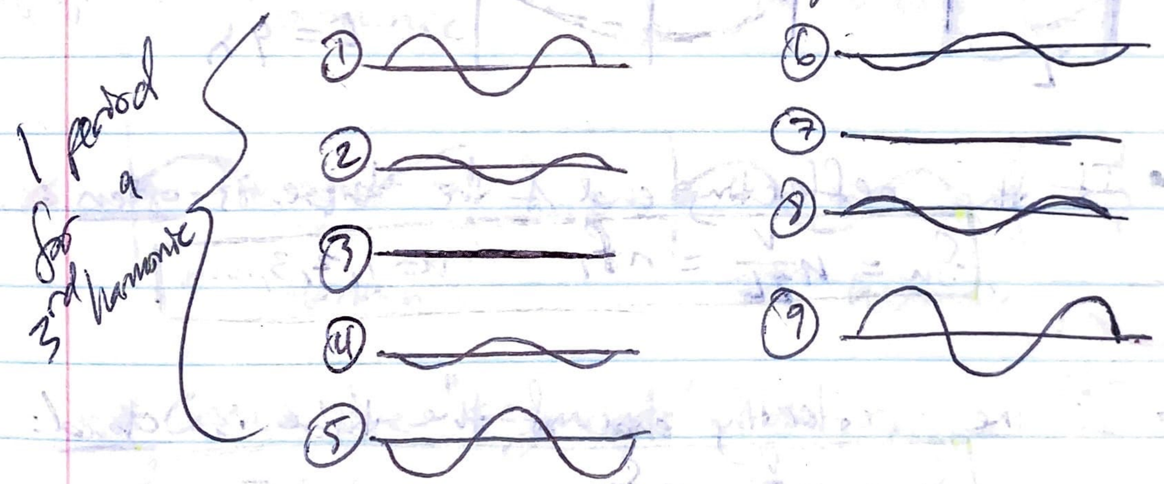
← Wave inverted
↕ Flips

- Standing waves → pattern that results when 2 waves of same f, λ, A travel in opposite directions & interfere
  - Have nodes & antinodes
    - ↳ maintains 0 displacement
    - ↳ antinode → halfway between (largest Δd) → at amplitudes

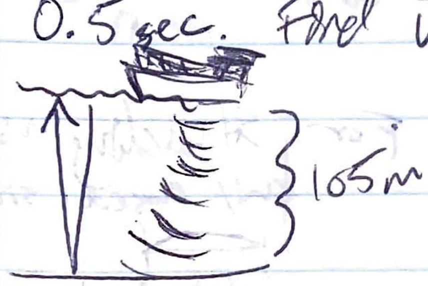


- Altering a system in Harmonic Motion
  - Positive force on system → add energy
  - Resonance (natural frequency)
    - ↳ when f of force applied to system matches natural frequency of vibration of system resulting in a large amplitude of vibration

- <u>Standing Waves</u> (cont...) → also known as stationary waves; a wave in a medium in which each point on the axis of the wave has an associated constant amplitude.



$\boxed{\text{Ex. 1}}$ Sonar/radar on a ship sent out a wave f @ 500 Hz down to the bottom of the sea floor (105 m) } it took 0.5 sec. Find $\vec{v}$ of waves.



Δx = waves go down then back up

$\Delta x = 210$ m

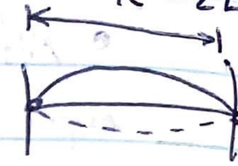
$t = 0.5$

$V = \frac{\Delta x}{t}$

$$\boxed{V = \frac{210}{0.5}}$$

- Frequencies of a standing wave

$\lambda = 2L$

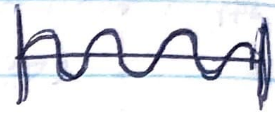


f_1 = fundamental f (1st Harmonic)

$\lambda = L$

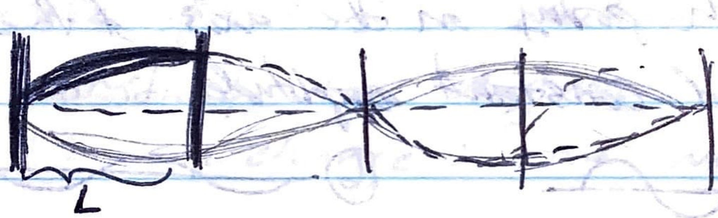
f_2 = 2nd harmonic $\quad f_2 = 2 f_1$

f_n = nth harmonic $\quad \boxed{f_n = n f_1 \quad n = 1, 3, 5, 7 \ldots}$



$V = f_1 \lambda_1 = f_1 2L \qquad f_1 2\cancel{L} = f_2 \cancel{L}$

$= f_2 \lambda_2 = f_2 L \qquad 2 f_1 = f_2$



E.P.

$\lambda = \frac{1}{4} L \qquad \lambda$ = 1 full "cycle"

$L = 4\lambda$

- If the reflecting end of the tube is open:

open

$$\boxed{f_n = n \frac{V}{2L} = n f_1 \qquad n = 1, 2, 3, \ldots}$$

- If the reflecting end of the tube is closed:

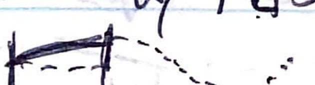
closed

$$\boxed{f_n = n \frac{V}{4L} = n f_1 \qquad n = 1, 3, 5, \ldots}$$

The frequencies $f_1, 2f_1, 3f_1$, and so on form a harmonic series

Next Page cont.

For standing waves w/ open side:



1st harmonic

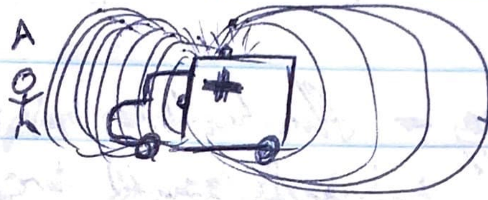
For standing waves w/ closed side

1st harmonic

• Doppler Effect)))))

- is an observed change in f when there is a relative motion btwn source of waves & receiver

higher pitch & f



lower pitch & f

*AKA Doppler shift

- effects sound & light

•

3rd harmonic

1st harmonic

n=1,3,5,7,...

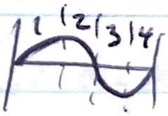
only odd harmonies

5th harmonic

7th harmonic

} so forth

* Count the "segments" of the wave



•

2nd harmonic (1 λ)

1st harmonic

n=1,2,3,4,...

•

1st harmonic

2nd harmonic

- Amplitude on a wave is due to the energy in the system. (More energy = greater amplitude)

- <u>Beat frequency</u> → fluctuations in a sound of a wave when 2 frequencies (different) are played together

$$\boxed{f_b = |f_1 - f_2|}$$

- $\lambda_1 f_1 = \lambda_n f_n$

$$2L(\cancel{f_1}) = \lambda_n n \cancel{f_1}$$

$$\boxed{\left(\frac{2}{n}\right)L = \lambda_n}$$

- $$\boxed{E = A^2}$$

Ch. 7

Rotational Motion / Kinematics

- Radian measure $180° = \pi$ rad
  - angular quantities in Physics → expressed in radians

- Angular Displacement
  - difference ~~~~ in its final & initial ∡s

$$\boxed{\Delta\theta = \theta_f - \theta_i}$$ SI: radian (rad)

- Average Angular Speed (ω_{av})

$$\boxed{\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}}$$ SI: (rad/s)

  - Over a long period of time

- Instantaneous ∡ speed ω (very short time interval)

$$\boxed{\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}}$$ SI: (rad/s)

* ω → ⊕ when rotating counterclockwise ↺ (θ is increasing)
* ω → ⊖ when rotating clockwise ↻ (θ decreasing)
- When ∡ speed is constant, the instantaneous ∡ speed is equal to ω_{AVG}

- Average Angular Acceleration (α_{av})

$a_{av} \Rightarrow \alpha_{av}$

$$\boxed{a_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}}$$ SI: (rad/s^2)

  - change in ∡ speed over time
  - Signs apply depending on direction of motion

- Instantaneous ∡ acceleration (α)

$$\boxed{\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}}$$

SI: same as α_{av}

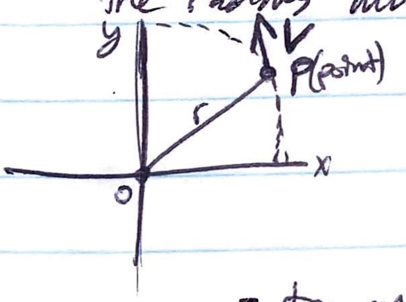
- Comparing linear / ∡ motion equations

Linear	Rotational
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$

- v replaced w/ ω
- a replaced w/ α
- Δx replaced w/ $\Delta\theta$

$a \rightarrow \alpha$ or a_c

- Tangential Speed
  - speed of a point on a rotating object equals the radius multiplied by angular speed



* The direction of P's velocity is tangent to the circular path

- tangential & linear speed are the same ($\vec{v}$ is used)

$\boxed{v = \omega r}$ or $\boxed{\omega = \frac{v}{r}}$

v = linear/tangential $\vec{v}$
r = radius
ω = angular speed

$$\boxed{S = r\theta}$$

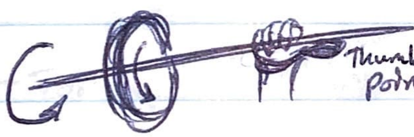
S → arc length
r → radius

- A relationship between linear / rotational quantities

$$\boxed{\begin{aligned} V &= r\omega \\ a &= r\alpha \end{aligned}}$$ ← How to convert liner & rotational

- All angular quantities are vectors

- Right Hand Rule



$\omega \rightarrow$

Thumb points to direction of $\vec{\omega}$

* Fingers indicate the direction of rotation

Torque & Center of Mass

- Torque → is produced by a turning force & tends to produce rotational acceleration

τ ↙

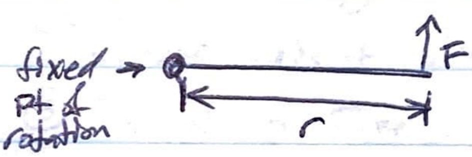
AKA: changes rotational motion

- Torque (τ) is the tendency of a force to rotate an object some axis

$$\boxed{\tau = r\,F_y}$$

τ = tow = torque
r = distance
** F = Net force

SI: N·m



- Torque is a vector
  * direction is ⊥ to plane of rotation
  * Similar to rotational direction

- If Net torque is zero, the rate of rotation doesn't change
- Only the component of the force perpendicular to object will cause it to rotate / cause torque

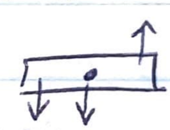
← y component matters (only) $F\sin\theta = F_y$

- Net torque (sum of all torques taking into account the direction of each)

- In order to be in Equilibrium
  - $\sum \vec{F} = 0$
  - $\sum \vec{\tau} = 0$

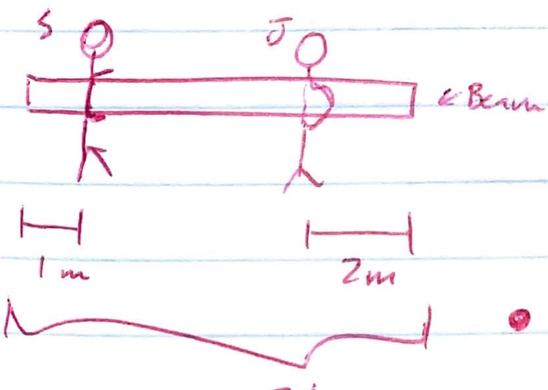
Both Force & τ are 0

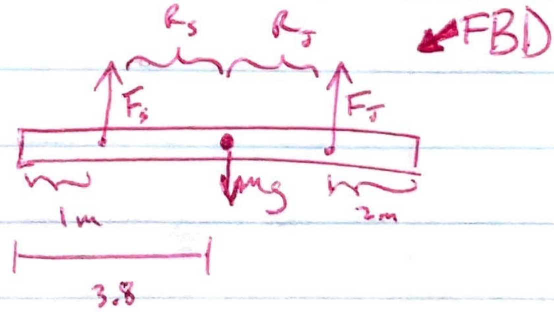
- Equilibrium Problems
  - A rigid object can be balanced by a single force equal in magnitude to weight of object (up force = down force) → no $\vec{a}$ on object
  - Zero net torque (object is not rotating or is rotating @ constant velocity, ω)
  - Solving Equilibrium problems
    1. Draw Diagram
    2. Choose axis of rotation (so 1 of the torques = 0)
    3. Draw a Free body diagram (more detailed)

Ex: Pulley  IOR 

4. Write the τ(net) equation } set it to 0 (zero)
5. Write F(net) equation(s) } set them each to 0
6. Then solve

Ex. 1





$mg = 950\ N$

$R_S = 2.8$

$R_J = 1.8$

- $\tau_{net} = -F_S r_S + mg(0) + F_J r_J = 0$

$F_S + F_J = mg$

- $F_{net} = F_S + F_J - mg = 0$

$-F_S(2.8) + F_J(1.8) = 0$

$F_J = \dfrac{F_S(2.8)}{1.8}$

$F_S + F_S(1.6) - 450 = 0$

$\boxed{F_S = 173.07\ N}$

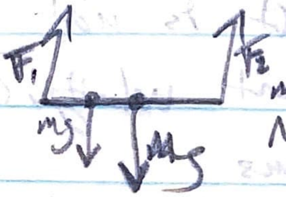
Ex. 2 Window washer is standing on a scaffold supported by 2 vert. ropes. Weight of scaffold is 200 N & is 3 m long. What is tension in each rope when the 700 N worker stands on it 1 m from edge.

Axis of symm. →

$\tau_{net} = 0 - 1(mg) - 1.5(Mg) + 3F_2 = 0$

$F_{net} = F_1 - mg - Mg + F_2 = 0$

FBD:



$mg = 700$ N
$Mg = 200$ N

$\tau_{net} = 0 - 1(700) - 1.5(200) + 3F_2 = 0$

$\tau_{net} = -1000 + 3F_2 = 0$

$F_2 = \frac{1000}{3} = \boxed{333\text{ N}}$

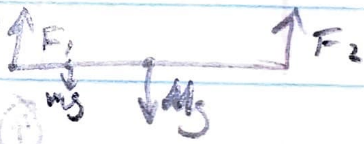
$F_{net} = F_1 - 900 + F_2 = 0$

$= F_1 - 900 + \left(\frac{1000}{3}\right) = 0$

$\boxed{F_1 = 567\text{ N}}$

Axis of symm →

FBD



$\tau_{net} = -F_1(1.5) - mg(0.5)$

Linear		Angular
Δx	$=$	$r(\Delta\theta)$
v	$=$	$r(\omega)$
a	$=$	$r(\alpha)$

r = radius

Relationship btwn them

$\vec{a} = \frac{\vec{F}}{m}$ $I \rightarrow$ rotational inertia (AKA moment of Inertia)

* Ex: It is harder to spin w/ dumbells in hands w/ arms extended than arms not extended

$\boxed{\alpha = \frac{\tau}{I}}$ or $\tau = I\alpha$

To find Moment of Inertia:

- Loop (⊛) $I = mr^2$ ← And for point mass inertial rotation
- Disc $I = \frac{1}{2}mr^2$

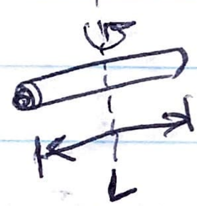
* Moments & Inertia for various rigid objects

Hoop/ thin shell

$I = MR^2$

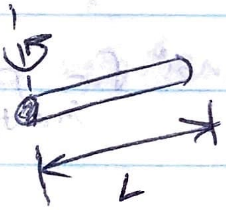
Solid cylinder

$I = \frac{1}{2}MR^2$



long thin rod w/ rotation of axis through center

$I = \frac{1}{12}ML^2$



long thin rod w/ rotation axis through end

$I = \frac{1}{3}ML^2$

Solid sphere

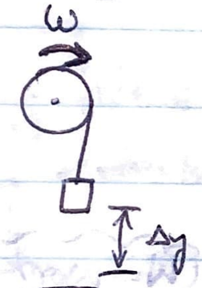
$I = \frac{2}{5}MR^2$

Thin spherical shell

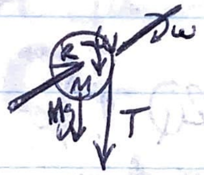
$I = \frac{2}{3}MR^2$

- Solving Non-equilibrium Problems 
  - Find the equation for I based on axis of rot./shape
  - Write τ_{net} equations & set it to $I\alpha$
  - Find α kinematically
  - F_{net} equations set equal to ma
  - $a = r\alpha$ (w/o "slipping")
  - be careful about signs for α & a

Ex. 1



Reel



Bucket

cylindrical reel $\rightarrow I = \frac{mr^2}{2}$

$\tau_{net} = -TR = I\alpha$

$F_{net\,y} = T - mg = ma$

$R = 0.4\,m$

$M = 3.00\,kg$

$m = 2.00\,kg$

$T = ?$

$a = ?$

$V_0 = 0\,m/s$

$t = 3\,sec$

$\Delta y = ?$

$-TR = I\alpha$

$T = \frac{I\alpha}{-R}$

$T = \left(\frac{MR^2}{2}\right)\frac{a/R}{R}$

$T = -\frac{Ma}{2}$

$T - mg = ma$

$T = ma + mg$

$-\frac{Ma}{2} = ma + mg$

$a = \frac{-mg}{(M/2 + m)}$

$\boxed{a = -5.60\,m/s^2}$

① use τ_{net} / F_{net} to get 2 equations w/ 2 unknowns (T & a)

② Use $a = \alpha r$ to tie them together

③ Use kinematics to find Δy

Ex. 2

- 150 kg Merry-go-round
- Disc $\Longrightarrow$ $I = \frac{mr^2}{2}$
- Radius → 1.5 m
- $\omega_0 = 0$
- $\omega_f = 0.5 \frac{rev}{se} \cdot 2\pi = \pi \frac{rad}{sec}$
- $T = ?$

Strategy
- Find α w/ kinematics
- use $\tau_{net} = I\alpha$ to find T

$$\tau_{net} = rT = I\alpha$$

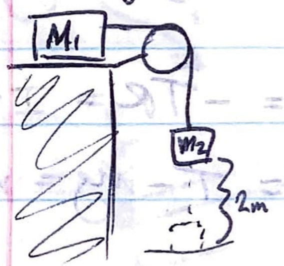
$$T = \frac{I\alpha}{r}$$

$$T = \left(\frac{mr^2}{2}\right)\left(\frac{\alpha}{r}\right)$$

$$T = \frac{mr\alpha}{2}$$

$$T = \frac{150(1.5)(0.5)}{2}$$

Ex. 1 Energy Q (FRQ 9.1)
using energy



W = work (Frictional force $\cdot \Delta x$)

$$mgh_0 + W = \underbrace{\tfrac{1}{2}m_1v^2}_{KE_t} + \underbrace{\tfrac{1}{2}m_2v^2}_{KE_t} + \underbrace{\tfrac{1}{2}I\omega^2}_{KE_R}$$

$$mgh_0 + ((-F_{fk})(2)) = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 + \tfrac{1}{2}I\omega^2$$

- $M_1 = 5\text{ kg}$
- $M_2 = 7\text{ kg}$
- M_2 drops 2m

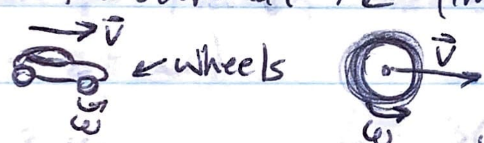
- Rotational KE

  * $\boxed{KE_r = \frac{1}{2} I \omega^2}$ SI: Joules

- add work to add energy
  - ⊕ if added
  - ⊖ if taken out
- Conservation of ME

$$\overbrace{KE_{t_0} + KE_{r_0} + PE_0 + W}^{\text{Initial}} = \overbrace{KE_{tf} + KE_{rf} + PE_f}^{\text{Final}}$$

KE_t → translational KE (linear motion/energy)



↳ refers to the center of mass's motion

For solving:
- use $v = r\omega$ to combine terms when solving (only when "no slipping" occurs)
- substitute the correct expression for I

- Work could help/inhibit energy
  Ex: Friction ($W = F_{fk} \cdot \Delta x$)

- Angular Momentum ("L" not p)

$$\boxed{L = I\omega}$$

* L depends on axis of rotation (how it's spinning)

$L = I\omega$

$L = (kg \cdot m^2)(rad/s)$ OR SI unit: $\frac{kg\, m^2}{s}$ OR $N \cdot m \cdot s$

[radians are unitless]

Ex. 1 1 revolution per 4 seconds means $\frac{2\pi}{4} = 1.57 \frac{rad}{s}$

(1 rev = 2π rad)

Mass = 3 kg

r = 2 m

$L = 3(2)^2(1.57) = 18.8 \frac{kg\, m^2}{s}$

- τ L is changed by applying a $\Sigma\tau$ for some period of time

$\Delta p = F(\Delta t) = I$

τ version of impulse → $\Delta L = \tau(\Delta t)$

- Conservation of L
  - If there isn't any outside τ then the L remains constant $L_o = L_f$

Ex. 2

$I_o = 2\ kg \cdot m^2$

$I_f = 2.5\ kg \cdot m^2$

$\omega_o = 8$ rad/s

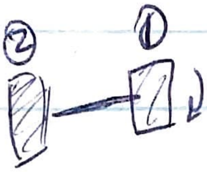
$\omega_f = ?$

$L_o = L_f$

$2(8) = 2.5(\omega_f)$

$$\boxed{\omega_f = 6.4 \frac{rad}{s}}$$

Ex.3



$\omega_0 = 10 \frac{rad}{s}$

$I = 1\ kg\,m^2$

$\omega_0 = 0 \frac{rad}{s}$

• ② moves until it hits #①

$I_o = I_f$

$\boxed{\omega\ I}\ I_1\omega_{01} + \cancel{I_2\omega_{02}} = I_1\omega_f + I_2\omega_f$

$I_f = 1+1 = 2\ kg\cdot m^2$

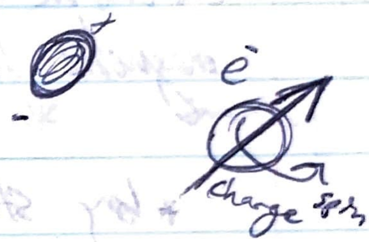
after they touch

$\cancel{I}\omega_0 = 2\cancel{I_1}\omega_f$ *

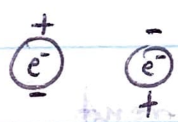
$\frac{\omega_0}{2} = \omega_f$

$\omega_f = \frac{10}{2} = \boxed{5}\ rad/s$

Magnetism 

- magnetism & electricity are "married together"
- It is caused by moving charge and vice versa
    - Ways charge can move:
        • spin 
        • current 
- Ferromagnetism
    - atoms are baby magnets 

    - moving charge
        ↳ magnetic charge/field
    - For most elements, the fields cancel

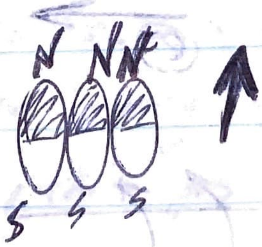


← cancel out

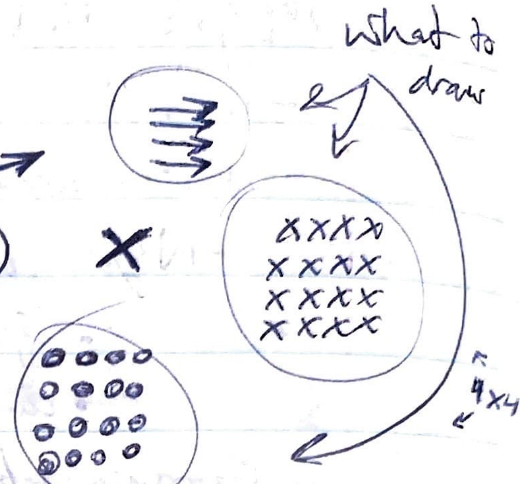
$\boxed{\text{OR}}$

    - Imbalance : 

- Magnetic domain
    - <u>Domain</u>: fields of groups of atoms aligned in same way
    - representation: arrow (vectors)

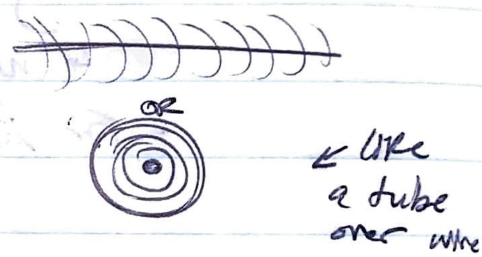
Domains 

- 3D Field Representation
  - side orientation
  - Field leaving (travelling away) X
  - Coming at you o



- current in a wire causes magnetic field

* long straight current

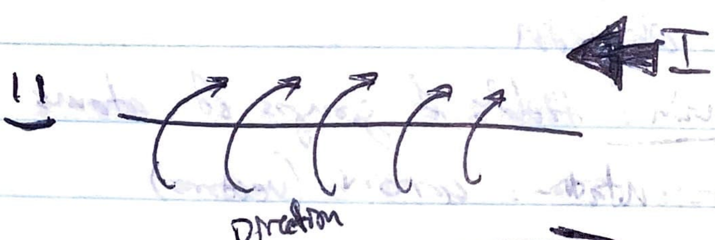


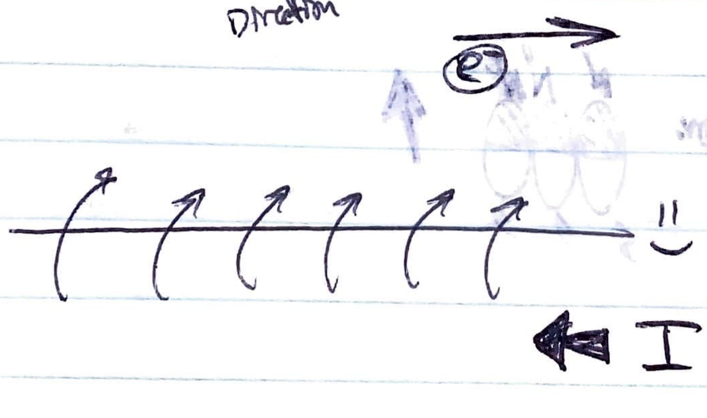
- current convention
  - convention for current

    I → (+) →

  - Electrons

    ← (e) (+) current (+ charge) →





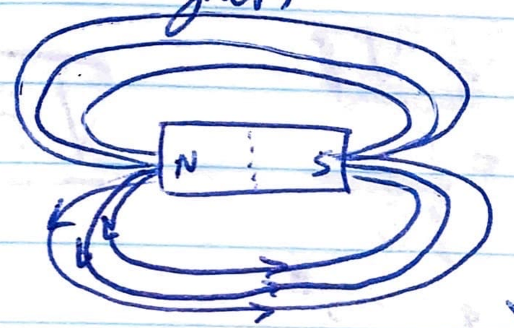
- curling Right Hand Rule

⊕ shows direction of current

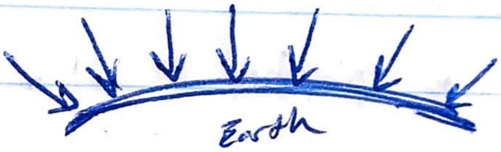
↑ negative charge side (e^-)

- curling fingers indicate direction of flowing current around a magnetic wire

- Bar Magnets



Earth is like a bar magnet



wire in a circular config.

⇒

like a bar magnet

- Magnetic Force
  * Have to have a charged particle (+ or −)
  * Must be moving (charged particle)
  * Charged particle goes through magnetic field

- Velocity is parallel to field ⇒ doesn't experience F
- If it is ⊥ (or ⊥-component) ⇒ will

Magnitude of Force

$$\boxed{F_B = qvB}$$

B= magnetic field strength (In teslas)
V= velocity
q= charge

- F_B is maximum when V is $\perp$

Ex.

$q = +1.6 \times 10^{-19}$ C
$F_B = 8.8 \times 10^{-11}$ N
$B = 5.5 \times 10^{-5}$ T
V= ? m/s East

$\boxed{\text{Find V}}$

$V = \frac{F_B}{qB}$

$V = \frac{8.8 \times 10^{-11}}{(1.6 \times 10^{-19})(5.5 \times 10^{-5})}$

$\boxed{V = 100{,}000 \text{ m/s}}$

$\boxed{F_B\ B\ \text{up}}$

- current carrying wire in external magnetic field experiences a Force

$$\boxed{F_B = BI\ell}$$

Ex.

Wire 36 m
I = 22 A
F = down = 4×10^{-2} N
B = ? ?

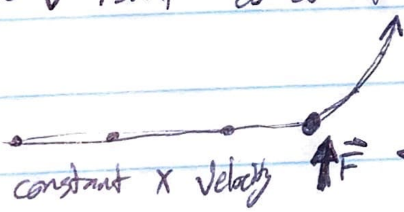
$B = \frac{F_B}{I\ell}$

$\boxed{B = 5 \times 10^{-5}}$ West

AP Exam Review

- Forces
  - When is there force?
    - not in equilibrium, object w/ mass is $\vec{a}$-ing
    - pushes/pulls ; field force ; electrical fields
  - Forces are applied, not given (exist only during contact (or field contact))
  - constant velocity = 0 (zero) Net Force ($ma = 0$)
  - Free fall:
    - $\vec{V}$ isn't constant however $\vec{F}$ & $\vec{a}$ is the same

EX: IN SPACE:



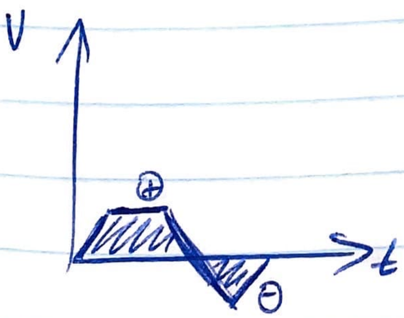
← backwards parabolic path

← constant y $\vec{a}$

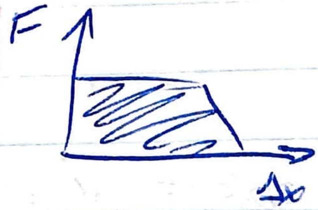
- Graphing
  - Determine slope
  - value of the graph
  - area under curve
  - use intercepts

Δx vs t
F vs Δx
τ vs α
V vs t
F vs t
T vs l
a vs t
p vs t

} Ex of graphs

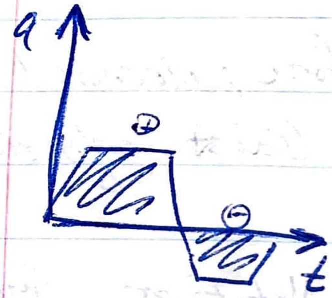


slope: $\vec{a}$ area: Δx



$F \cdot \Delta x$

$N \cdot m$ = Joule (Work) (or τ)



Area:

$a \cdot t = \frac{m}{s^2} \cdot s = \boxed{\frac{m}{s}}$

• Electricity

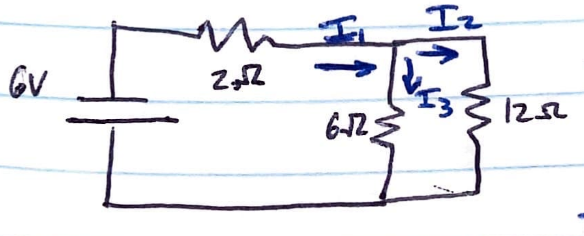
- Coulomb's Law (Electrical Forces)
  - ↳ charge is measured in coulombs (q)
  - ↳ charge on 1 electron $e = 1.6 \times 10^{-19}$ C
- Current (rate @ which charge is transferred)
  - ↳ $I = \frac{q}{t}$ $\left(\frac{\text{coulombs}}{\text{second}}\right)$ 1 Ampere = $\frac{1\,C}{1\,sec}$
- Voltage → Electrical Potential
  - ↳ PE_e per unit of charge (coulomb)
    - Joules/Coulombs
- Resistance → resistance to current
  - ↳ depends on shape (ℓ & A), material, temperature
  - ↳ the higher the temp, the larger the resistance
- Ohm's law (Voltage & R. is proportional)

  $I = \frac{V}{R}$ or $V = IR$ ← To find Voltage drop

  ↳ Equivalent Resistance (R_{eq}) → "What battery sees" (Total)

  $I_{CKT} = \frac{V_{battery}}{R_{eq}}$

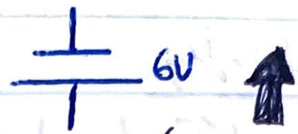
  "the higher the R, the smaller the battery push"
- Kirchhoff's Laws (conservation of charge)



* current coming into a junction is equal to current out of both ends of junction

Junction Rule: $I_1 - I_2 - I_3 = 0$ OR $I_1 = I_2 + I_3$

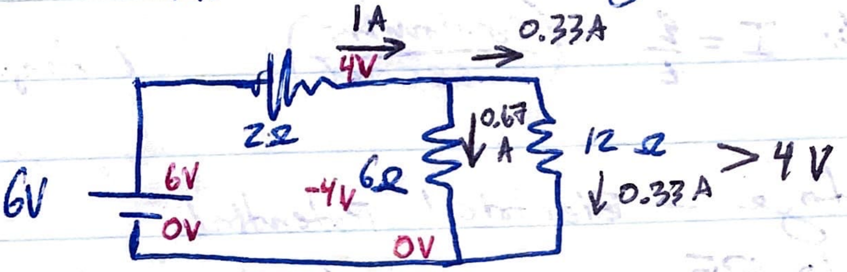
↳ energy is conserved }

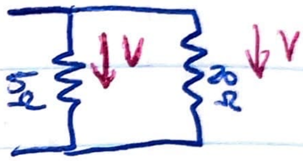


"As you go thru battery, you gain 6 V"

- the amount of energy used/ delivered in a loop has to equal the amount gained.

- AKA: Kirchhoff's Voltage Law





Voltage drops into parallel are equal even w/ diff. Ω

- Power

↳ rate @ which energy is delivered

$\boxed{P = IV}$ or $\boxed{P = \frac{V^2}{R}}$ or $\boxed{P = I^2 R}$

- Moving in a Uniform Circular Path
  - even if @ constant SPEED, the Velocity isn't so it is accelerating

* Velocity's direction is constantly changing

← force that "course corrects" to maintain a circular path

→ Centripetal Force (center seeking F)

$$F_{net} = ma_c = \frac{mv^2}{r}$$

$$a_c = \frac{v^2}{r}$$

- Rotational motion
  - equilibrium → $\tau_{net} = 0$
  - non-equilibrium → $\tau_{net} = I\alpha$